# Solutions to EA-1 Examination
# Spring, 2001

## Question 1

$1 - d^{(m)}/m = (1 - d^{(2m)}/2m)^2$

Substituting the given values of $d^{(m)}$ and $d^{(2m)}$,

$$1 - \frac{.085256}{m} = \left(1 - \frac{.085715}{2m}\right)^2 \quad \Rightarrow \quad 1 - \frac{.085256}{m} = 1 - \frac{.171430}{2m} + \frac{.001837}{m^2}$$

(multiplying the equation by $m^2$) $\Rightarrow$ $m^2 - .085256m = m^2 - .085715m + .001837$

$\Rightarrow$ $m = 4$

Therefore, $d^{(4)} = .085256$, and $\dfrac{d^{(4)}}{4} = .021314$

Recall that $i = \dfrac{d}{1-d}$

$$i^{(4)}/4 = \frac{\frac{d^{(4)}}{4}}{1 - \frac{d^{(4)}}{4}} = \frac{.021314}{1 - .021314} = .021778$$

$i^{(12)}/12 = 1.021778^{\text{♋}} - 1 = .007207$

$1000i^{(3m)} = 1000i^{(12)} = 1000 \times 12 \times .007207 = 86.484$

Answer is C.

## Question 2

If the frequency of compounding is bi-monthly (every two months), then there are 6 interest periods in a year, and the effective monthly interest rate is 2.5% (15% divided by 6). The accumulated balance of the loan after 2 years would be:

$\$1,800 \times 1.025^{12} = \$2,420.80$

Therefore, the frequency of compounding is every two months.

Answer is B.

Note: Each possible frequency can be checked until the correct one is determined as above.

## Question 3

Original loan payment = $\dfrac{100{,}000}{a_{\overline{30}|.06}}$ = 7,264.89

The equation of value representing the present value of the loan payments (other than the smaller final repayment) is:

$$100{,}000 = 5{,}000v^5 + 5000v^{10} + 7{,}264.89\, a_{\overline{N}|.06}$$
$$= 5{,}000v^5 + 5000v^{10} + (7{,}264.89)\left(\frac{1 - v^N}{.06}\right)$$

Solving for $v^N$:

$v^N = .228026$

Solving for N:

$\ln(v^N) = \ln(.228026) \quad \Rightarrow \quad N \times \ln(v) = \ln(.228026)$
$\quad \Rightarrow \quad N = 25.37$

Therefore, the final repayment will be at the end of 26 years.

If F is the final repayment,

$$100{,}000 = 5{,}000v^5 + 5000v^{10} + 7{,}264.89\, a_{\overline{25}|.06} + Fv^{26} \qquad \Rightarrow \qquad F = 2{,}739.00$$

Total payments under the original repayment schedule = 7,264.89 × 30 = 217,946.70

Total payments under the revised loan schedule =
(7,264.89 × 25) + 5,000 + 5,000 + 2,739 = 194,361.25

Total interest saved = 217,946.70 - 194,361.25 = 23,585.45

Answer is B.

## Question 4

The equation of value representing the present value of the loan payments is:

$$
\begin{aligned}
100{,}000 &= 3{,}100\, a_{\overline{110}|.03} + Xv^{111} + (2X)v^{112} + (2^2X)v^{113} + \ldots + (2^9X)v^{120} \\
&= 3{,}100\, a_{\overline{110}|.03} + v^{110}X(2v + (2v)^2 + (2v)^3 + \ldots + (2v)^{10}) \\
&= 3{,}100\, a_{\overline{110}|.03} + v^{110}X\left(\frac{2v - (2v)^{11}}{1 - 2v}\right)
\end{aligned}
$$

$X = 21.98$

Final repayment $= 21.98 \times 2^9 = 11{,}253.76$

Answer is B.

## Question 5

Recall the following formula for a geometric series:

$$1 + x + x^2 + \ldots + x^{n-1} = \frac{1 - x^n}{1 - x}$$

The present value of the contributions must equal the present value of the withdrawals. Let W be equal to the first annual withdrawal.

$$2{,}000 + 2{,}000(.97v) + \ldots + 2{,}000(.97v)^{19} = Wv^{23} + 1.04Wv^{24} + \ldots + 1.04^{24}Wv^{47}$$

$$2{,}000 \times \frac{1 - (.97v)^{20}}{1 - (.97v)} = Wv^{23} \times \frac{1 - (1.04v)^{25}}{1 - (1.04v)}$$

$W = 2{,}869.30$

The sum of the 1/1/2025 and 1/1/2026 withdrawals is:

$2{,}869.30 + (2{,}869.30 \times 1.04) = 5{,}853.37$

Answer is D.

## Question 6

The quarterly effective rate of interest is:

$i^{(4)}/4 = (1 + i^{(2)}/2)^{1/2} - 1 = 1.04^{1/2} - 1 = .019804$

The original loan payment is:

$$\frac{25{,}000}{a_{\overline{32}|.019804}} = 1{,}062.23$$

Outstanding balance after 10 payments = $1{,}062.23\, a_{\overline{22}|.019804} = 18{,}795.55$

Since the 11th and 12th repayments are not made, the outstanding balance after the 12th scheduled repayment is:

$18{,}795.55 \times 1.019804^2 = 19{,}547.38$

The present value of the final 20 repayments is represented by:

$$\begin{aligned}
19{,}547.38 &= X\, a_{\overline{20}|.019804} + (200v^2 + 200v^3 + 400v^4 + 400v^5 + \ldots + 1{,}800v^{19} + 2{,}000v^{20}) \\
&= X\, a_{\overline{20}|.019804} + (200v^2 + 400v^4 + \ldots + 2{,}000v^{20}) \\
&\qquad + (200v^3 + 400v^5 + \ldots + 1{,}800v^{19}) \\
&= X\, a_{\overline{20}|.019804} + 200v^2(1 + 2v^2 + \ldots + 10v^{18}) + 200v^3(1 + 2v^2 + \ldots + 9v^{16}) \\
&= X\, a_{\overline{20}|.019804} + 200\, v^2_{.019804}\, (I\ddot{a})_{\overline{10}|.04} + 200\, v^3_{.019804}\, (I\ddot{a})_{\overline{9}|.04} \\
&= X\, a_{\overline{20}|.019804} + \left(200\, v^2_{.019804} \times \frac{\ddot{a}_{\overline{10}|.04} - 10v^{10}}{(.04/1.04)}\right) + \left(200\, v^3_{.019804} \times \frac{\ddot{a}_{\overline{9}|.04} - 9v^9}{(.04/1.04)}\right)
\end{aligned}$$

Note that the interest rate used for the increasing annuities is 4%, which is the effective semiannual interest rate, since the payments occur once every 6 months.

X = 258.72

Answer is C.

## Question 7

A = \$1,700 + \$1,900 + \$2,100 + \$2,300 + \$2,500 = \$10,500

B = $\$1{,}700v^2 + \$1{,}900v^4 + \$2{,}100v^6 + \$2{,}300v^8 + \$2{,}500v^{10} = \$7{,}337$

A – B = \$10,500 - \$7,337 = \$3,163

Answer is C.

## Question 8

The principal portion of any repayment is equal to the repayment multiplied by $v^N$, where N is the number of remaining payments, including the current payment.

The principal repayment in years 1 and 2 is:

$\$1{,}000 \times (v^4 + v^3)$

Similarly, the interest portion of any repayment is equal to the repayment multiplied by $(1 - v^N)$, where N is the number of remaining payments, including the current payment.

Therefore, the sum of the interest repayments in years 3 and 4 is:

$\$1{,}000 \times ([1 - v^2] + [1 - v])$

Therefore, based upon the information given in the question,

$\$1{,}000 \times (v^3 + v^4) = 10v^2 \times \$1{,}000 \times ([1 - v^2] + [1 - v])$

$v^3 + v^4 = 20v^2 - 10v^3 - 10v^4$

$v + v^2 = 20 - 10v - 10v^2$

$11v^2 + 11v - 20 = 0$

Recall the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In this case, a = 11, b = 11, and c = -20.

Substituting,

$$v = \frac{-11 \pm \sqrt{11^2 - (4)(11)(-20)}}{(2)(11)} = .938$$

Answer is C.

## Question 9

The effective rate of interest every three years is:

$1.04^3 - 1 = .124864$

The loan payment is:

$\$1{,}000 / a_{\overline{10}|.124864} = \$180.52$

The principal portion of any repayment is equal to the repayment multiplied by $v^N$, where N is the number of remaining payments, including the current payment. The principal portion of the fifth repayment (note that there are 6 remaining payments immediately prior to the fifth repayment being made) is:

$\$180.52 \times v^6 = \$89.11$

Answer is B.

## Question 10

Amortized value represents the present value of all future bond payments (coupons and redemption amount). The amortized value on January 1, 2002 includes the present value of all coupon payments made during 2002 and later, as well as the present value of the redemption amount. The amortized value on July 1, 2001 is equal to the present value of the January 1, 2002 amortized value, plus the present value of the coupon payment made on December 31, 2001.

$$13{,}741.11 = 13{,}629.67v + 400v \quad \Rightarrow \quad v = .979432$$
$$\Rightarrow \quad i = 1/v - 1 = .021$$

If R is the redemption amount,

$$\$13{,}629.67 = Rv^{14} + 400\,a_{\overline{14}|.021} \quad \Rightarrow \quad R = \$11{,}800$$

Answer is C.

## Question 11

The amortized value of the bond as of January 1, 2005 is equal to the accumulation of the original purchase price, less the accumulated value of the coupon payments. Coupons were paid on December 31, 2002 and December 31, 2004.

A = Amortized value on January 1, 2005 = $(691.49 \times 1.08^4) - (60 \times 1.08^2) - 60 = 810.78$

The amortized value of the bond as of January 1, 2007 is equal to the accumulation of the January 1, 2005 value, less the coupon paid on December 31, 2006.

B = Amortized value on January 1, 2007 = $(810.78 \times 1.08^2) - 60 = 885.69$

$B - A = 885.69 - 810.78 = 74.91$

Answer is E.

## Question 12

The monthly interest rate for the initial loan payment is:

$.07/12 = .00583333$

Initial loan payment = $\$200{,}000 / a_{\overline{360}|.00583333} = \$1{,}331$

The outstanding balance after the second year is:

$\$1{,}331\, a_{\overline{336}|.00583333} = \$195{,}848$

The revised loan payment using the 7.5% interest rate (.625% per month) is:

$\$195{,}848 / a_{\overline{336}|.00625} = \$1{,}396$

The outstanding balance after the fourth year is:

$\$1{,}396\, a_{\overline{312}|.00625} = \$191{,}388$

The revised loan payment using the 8% interest rate (.666667% per month) is:

$\$191{,}388 / a_{\overline{312}|.00666667} = \$1{,}460$

The total payment over the term of the loan is:

(\$1,331 × 24) + (\$1,396 × 24) + (\$1,460 × 312) = \$520,968

The total interest paid over the term of the loan is equal to the difference between the total payments and the original loan amount:

\$520,968 - \$200,000 = \$320,968

Answer is D.

## Question 13

The present value of the perpetuity is:

$\$1.00 \times a_{\overline{\infty}|} = 1/i = 1/.06 = 16.6667$

The duration is equal to the ratio of the time-weighted present value of the payments to the present value of the payments.

The duration for the perpetuity is:

$$\begin{aligned}\bar{d} &= [v + 2v^2 + 3v^3 + \ldots)]/16.6667 \\ &= (Ia)_{\overline{\infty}|}/16.6667 \\ &= [1/.06 + (1/.06)^2]/16.6667 \\ &= 17.6667\end{aligned}$$

Note that this is the MacCaulay duration. The modified duration can be determined as follows:

$\bar{v} = \bar{d}/(1+i) = 17.6667/1.06 = 16.6667$

Answer is A.

## Question 14

The present value of portfolio C is:

$PV_C = 10{,}000v^4 = 7{,}629$

The duration is equal to the ratio of the time-weighted present value of the payments to the present value of the payments. Note that this is the MacCaulay duration. The modified duration is determined by discounting the MacCaulay duration with interest for one year. Since the modified duration of each portfolio is the same, then the MacCaulay duration must also be the same for each portfolio. The MacCaulay duration for portfolio C is:

$\bar{d}_C = (4 \times 10{,}000\ v^4)/7{,}629 = 4$

Let R equal the redemption value of the 4-year bonds in portfolio A. Note that X is equal to the present value of the maturity value of the 5-year zero-coupon bond, since it is the amount invested.

The present value of portfolio A is:

$PV_A = .07R\,a_{\overline{4}|} + Rv^4 + X = 7{,}629$

$\Rightarrow \quad .07R\,\dfrac{1 - v^4}{.07} + Rv^4 + X = 7{,}629 \quad \Rightarrow \quad R + X = 7{,}629$

The MacCaulay duration for portfolio A is:

$\bar{d}_A = (.07Rv + (2)(.07R)v^2 + (3)(.07R)v^3 + (4)(.07R)v^4 + 4Rv^4 + 5X)/7{,}629 = 4$

$\Rightarrow \quad (.07R\,(Ia)_{\overline{4}|} + 4Rv^4 + 5X)/7{,}629 = 4$

$\Rightarrow \quad (.07R\,\dfrac{\ddot{a}_{\overline{4}|} - 4v^4}{.07} + 4Rv^4 + 5X)/7{,}629 = 4$

$\Rightarrow \quad R\,\ddot{a}_{\overline{4}|} + 5X = 30{,}516$

$\Rightarrow \quad 7{,}629\,\ddot{a}_{\overline{4}|} - X\,\ddot{a}_{\overline{4}|} + 5X = 30{,}516$ (substituting from the present value above)

$\Rightarrow \quad X = 2{,}083$

Similarly, the value of Y can be solved for portfolio B. Let S equal the redemption value of the bonds in portfolio B. The present value equation of value yields:

$S + X = 7{,}629$

The MacCauley duration equation of value yields:

$S\ddot{a}_{\overline{3}|} + 5Y = 30{,}516$

$\Rightarrow \quad 7{,}629\,\ddot{a}_{\overline{3}|} - Y\ddot{a}_{\overline{3}|} + 5Y = 30{,}516$

$\Rightarrow \quad Y = 4{,}149$

$Y – X = 4{,}149 - 2{,}083 = 2{,}066$

Answer is B.

## Question 15

Recall the following formula for a geometric series:

$$1 + x + x^2 + \ldots + x^{n-1} = \frac{1-x^n}{1-x}$$

In determining the present value X, the assumed increase in the CPI (6%) exceeds the stated 3 percentage points by 3%. The present value is:

$$X = 10{,}000 \times (v + 1.03v^2 + \ldots\ 1.03^{11}v^{12}) = 10{,}000v \times (1 + 1.03v + \ldots\ 1.03^{11}v^{11})$$
$$= 10{,}000v \times \frac{1-(1.03v)^{12}}{1-1.03v} = 86{,}762$$

In determining the present value Y, the assumed increase in the CPI (4%) exceeds the stated 3 percentage points by 1%. The present value is:

$$Y = 10{,}000 \times (v + 1.01v^2 + \ldots\ 1.01^{11}v^{12}) = 10{,}000v \times (1 + 1.01v + \ldots\ 1.01^{11}v^{11})$$
$$= 10{,}000v \times \frac{1-(1.01v)^{12}}{1-1.01v} = 78{,}932$$

$X – Y = 86{,}762 - 78{,}932 = 7{,}830$

Answer is B.

## Question 16

The last age at which there could be an annuity payment is age 99.

$$\$100a_{60} = \$100 \times (vp_{60} + v^2{}_2p_{60} + .. + v^{39}{}_{39}p_{60}) = \$100 \times (\tfrac{39}{40}v + \tfrac{38}{40}v^2 + .. + \tfrac{1}{40}v^{39})$$
$$= \$100 \times \tfrac{1}{40} \times (39v + 38v^2 + .. + v^{39}) = \$100 \times \tfrac{1}{40} \times (Da)_{\overline{39|}}$$
$$= \$2.50 \times \frac{39 - a_{\overline{39|}}}{.06} = \$1{,}002$$

Answer is B.

## Question 17

Note that the first payment occurs at age 107. Since $l_{110} = 0$, the only possible payments occur at ages 107, 108 and 109, with the payments at ages 107 and 108 guaranteed provided Smith survives to age 106.

$$X = vp_{105} \times (1{,}000\,a_{\overline{2|}} + 1{,}000v^3{}_3p_{106}) = .8v \times (1{,}000\,a_{\overline{2|}} + (1{,}000v^3 \times \frac{.6}{.8} \times \frac{.4}{.6} \times \frac{.2}{.4}))$$
$$= 1{,}504$$

Answer is A.

## Question 18

Each remaining mortgage payment is:

$\$50{,}000/a_{\overline{3|}} = \$18{,}360$

The mortgage payments are guaranteed to be paid (either by the mortgagee or by the insurer). The present value of the future mortgage payments to be paid by the mortgagee is:

$$\$18{,}360 \times (vp_{57} + v^2{}_2p_{57} + v^3{}_3p_{57}) = \$18{,}360 \times (vl_{58}/l_{57} + v^2l_{59}/l_{57} + v^3l_{60}/l_{57})$$
$$= \tfrac{\$18{,}360}{9{,}574} \times (9{,}541v + 9{,}505v^2 + 9{,}467v^3)$$
$$= 49{,}641$$

The present value of the future mortgage payments to be paid by the insurer is:

$50{,}000 - 49{,}641 = 359$

Answer is B.

## Question 19

The additional cost is equal to the difference between the present value of the 10-year annuity certain and the 10-year deferred life annuity payable from ages 65 through 75. This is:

$$\$50{,}000v^{10}\,{}_{10}p_{55} \times (\ddot{a}_{\overline{10|}} - (\ddot{a}_{65} - {}_{10}p_{65}\,\ddot{a}_{75}\,v^{10})) = 16{,}389$$

Answer is A.

Note that ${}_{10}p_{65} = {}_{20}p_{55}/{}_{10}p_{55} = .67826$

## Question 20

Recall the approximation:

$$\mathring{e}_x \approx e_x + \tfrac{1}{2}$$

Therefore, $e_{52} = \mathring{e}_{52} - \tfrac{1}{2} = 25.5$

Also,

$p_{50} = l_{51}/l_{50} = 98{,}000/100{,}000 = .98$ and ${}_2p_{50} = l_{52}/l_{50} = 95{,}550/100{,}000 = .9555$

Using the formula for $e_x$:

$$\begin{aligned} e_{50} &= p_{50} + {}_2p_{50} + {}_3p_{50} + {}_4p_{50} + \ldots = p_{50} + {}_2p_{50}\,(1 + p_{52} + {}_2p_{52} + \ldots) \\ &= p_{50} + {}_2p_{50}\,(1 + e_{52}) = .98 + [.9555 \times (1 + 25.5)] = 26.3 \end{aligned}$$

Therefore,

$$\mathring{e}_{50} \approx e_{50} + \tfrac{1}{2} = 26.8$$

Answer is C.

## Question 21

The number of lives that die between the ages of 20 and 50 is represented by:

$l_{20} - l_{50}$

The total lifetime of all of the lives age x is represented by:

$xl_x + T_x$

where $xl_x$ is the past lifetime and $T_x$ is the future lifetime.

So, the total lifetime of the lives currently age 20 is:

$20l_{20} + T_{20}$

The total lifetime of the lives currently age 50 is:

$50l_{50} + T_{50}$

The average age at death is the ratio of the total lifetime to the number of lives that die. Therefore, the average age at death for the lives that die between the ages of 20 and 50 is:

$$\frac{20l_{20} + T_{20} - (50l_{50} + T_{50})}{l_{20} - l_{50}}$$

Recall that the number of persons living at age 20 and older is equal to $T_{20}$ in a stationary population, and the number of persons living at age 50 and older is equal to $T_{50}$ in a stationary population. Substituting,

$$\frac{(20 \times 1{,}080) + 21{,}600 - (50l_{50} + 2{,}700)}{1{,}080 - l_{50}} = 33.3333 \quad \Rightarrow \quad l_{50} = 270$$

Recall that $\overset{\circ}{e}_x = T_x/l_x$

So, $\overset{\circ}{e}_{50} = T_{50}/l_{50} = 2{,}700/270 = 10$

Answer is C.

## Question 22

Recall that $e_x = p_x(1 + e_{x+1})$. So,

$e_{40} = p_{40}(1 + e_{41}) \quad \Rightarrow \quad p_{40} = .9593$
$e_{41} = p_{41}(1 + e_{42}) \quad \Rightarrow \quad p_{41} = .9529$
$e_{42} = p_{42}(1 + e_{43}) \quad \Rightarrow \quad p_{42} = .9524$

The probability that one of Smith and Brown will die in 2001 and the other will die in 2002 is:

$$(q_{40})(p_{41} \times q_{42}) + (q_{41})(p_{40} \times q_{41}) = (.0407)(.9529 \times .0476) + (.0471)(.9593 \times .0471)$$
$$= .003974$$

Answer is C.

## Question 23

Case 1: All three are alive 15 years from now.

$({}_{15}p_{20})({}_{15}p_{25})({}_{15}p_{30}) = (.95)^{15}(.90)^{15}(.85)^{15} = .008333$

Case 2: Only Smith and Brown are alive 15 years from now.

$({}_{15}p_{20})({}_{15}p_{25})({}_{5}p_{30} - {}_{15}p_{30}) = (.95)^{15}(.90)^{15}(.85^5 - .85^{15}) = .033991$

Case 3: Only Smith and Green are alive 15 years from now.

$({}_{15}p_{20})({}_{15}p_{30})({}_{5}p_{25} - {}_{15}p_{25}) = (.95)^{15}(.85)^{15}(.90^5 - .90^{15}) = .015565$

Case 4: Only Brown and Green are alive 15 years from now.

$({}_{15}p_{25})({}_{15}p_{30})({}_{5}p_{20} - {}_{15}p_{20}) = (.90)^{15}(.85)^{15}(.95^5 - .95^{15}) = .005584$

Total probability = .008333 + .033991 + .015565 + .005584 = .063473

Answer is B.

## Question 24

The first 10 payments are certain to be paid provided the annuitant remains alive. The last 10 payments are only paid if the annuitant is still alive and the spouse was alive 10 years earlier. The present value of the annuity is:

$$\$10{,}000 \times ( a_{65:\overline{10|}} + v^{11}\,{}_{11}p_{65}\,p_{60} + v^{12}\,{}_{12}p_{65}\,{}_{2}p_{60} + \ldots + v^{20}\,{}_{20}p_{65}\,{}_{10}p_{60})$$
$$= \$10{,}000 \times ( a_{65:\overline{10|}} + v^{10}\,{}_{10}p_{65}\; a_{60:75:\overline{10|}} ) = \$112{,}655$$

Answer is D.

## Question 25

According to statement (i):

${}_{10}p_{30:40:50} = .758 \quad \Rightarrow \quad {}_{30}p_{30} = .758$

According to statement (ii):

$(1 - {}_{5}p_{55})({}_{5}p_{50}) = .063 \quad \Rightarrow \quad {}_{5}p_{50} - {}_{10}p_{50} = .063 \quad \Rightarrow \quad {}_{25}p_{30} - {}_{30}p_{30} = .063\,{}_{20}p_{30}$

According to statement (iii):

$({}_{5}p_{30:35:40:45})(1 - {}_{5}p_{50}) = .045 \quad \Rightarrow \quad {}_{20}p_{30} - {}_{25}p_{30} = .045 \quad \Rightarrow \quad {}_{20}p_{30} = {}_{25}p_{30} + .045$

Substituting the results from statements (i) and (iii) into statement (ii):

${}_{25}p_{30} - .758 = .063({}_{25}p_{30} + .045) \quad \Rightarrow \quad {}_{25}p_{30} = .81199$

Answer is B.

## Question 26

From the given data,

$$q_{50}^{(d)} = 16/100 = .16$$

Using a standard approximation (see the London text, page 108) and taking into consideration that the deaths and withdrawals are uniform within the **single** decrement tables,

$$q_{50}^{\prime(d)} = \frac{q_{50}^{(d)}}{1-\frac{1}{2}q_{50}^{\prime(w)}} \quad \Rightarrow \quad q_{50}^{\prime(d)} = .16 \div (1 - (\tfrac{1}{2} \times .4)) = .2$$

And,

$$q_{50}^{\prime(w)} = \frac{q_{50}^{(w)}}{1-\frac{1}{2}q_{50}^{\prime(d)}} \quad \Rightarrow \quad q_{50}^{(w)} = q_{50}^{\prime(w)} \times (1-\tfrac{1}{2}q_{50}^{\prime(d)})$$

$$\Rightarrow \quad q_{50}^{(w)} = .4 \times (1 - (\tfrac{1}{2} \times .2)) = .36$$

Answer is A.

## Question 27

Using a standard approximation (see the London text, page 108),

$$q_{40}^{(2)} = \frac{q_{40}^{\prime(2)}}{1+\frac{1}{2}q_{40}^{\prime(1)}} = \frac{\frac{30}{100}}{1+\frac{1}{2}\times\frac{10}{100}} = .285714$$

Answer is E.

Here is an alternative solution.

Using the standard approximation $p_x^{(T)} = p_x^{'(1)} \times p_x^{'(2)} \times \ldots$ ,

$$p_{40}^{(T)} = .9 \times .7 = .63$$

And that means that $q_{40}^{(T)} = .37$.

Using another standard (but lesser known) approximation,

$$q'^{(2)}_{40} = 1 - (p^{(T)}_{40})^{\frac{d^{(2)}_{40}}{d^{(T)}_{40}}}$$

Dividing the d's by $q^{(T)}_{40}$,

$$q'^{(2)}_{40} = 1 - (p^{(T)}_{40})^{\frac{q^{(2)}_{40}}{q^{(T)}_{40}}}$$

$$.3 = 1 - (.63)^{\frac{q^{(2)}_{40}}{.37}}$$

$$.7 = (.63)^{\frac{q^{(2)}_{40}}{.37}}$$

$$\ln(.7) = \ln((.63)^{\frac{q^{(2)}_{40}}{.37}})$$

$$\ln(.7) = \frac{q^{(2)}_{40}}{.37}\ln(.63)$$

$$q^{(2)}_{40} = .28563$$

This still falls within range E.

## Question 28

Let x = the attained age of the pensioner, and y = the attained age of the spouse.

Setting the present value of the first two forms of payment equal to each other:

$$\$4{,}000a_x = \$3{,}600a_x + \$1{,}800(a_y - a_{xy}) \qquad \Rightarrow \qquad \frac{a_y - a_{xy}}{a_x} = \tfrac{2}{9}$$

Setting the present value of the first and third forms of payment equal to each other:

$$\$4{,}000a_x = \$3{,}582a_{xy} + \$1{,}791(a_y - a_{xy}) + \$4{,}000(a_x - a_{xy})$$

$$\Rightarrow \qquad \$4{,}000 = \$3{,}582\frac{a_{xy}}{a_x} + \$1{,}791\frac{a_y - a_{xy}}{a_x} + \$4{,}000\frac{a_x - a_{xy}}{a_x}$$

$$\Rightarrow \qquad \$4{,}000 = \$3{,}582\frac{a_{xy}}{a_x} + \$1{,}791\frac{a_y - a_{xy}}{a_x} + \$4{,}000 - \$4{,}000\frac{a_{xy}}{a_x}$$

$$\Rightarrow \qquad \$418\frac{a_{xy}}{a_x} = \$1{,}791 \times \tfrac{2}{9} \qquad \Rightarrow \qquad \frac{a_{xy}}{a_x} = .952153$$

Setting the present value of the first and fourth forms of payment equal to each other:

$\$4{,}000a_x = \$Ka_{xy} + (\$K/2)(a_y - a_{xy}) + (\$K/2)(a_x - a_{xy})$

$$\Rightarrow \quad \$4{,}000 = \$K\frac{a_{xy}}{a_x} + (\$K/2)\frac{a_y - a_{xy}}{a_x} + \$K/2 - (\$K/2)\frac{a_{xy}}{a_x}$$

$$\Rightarrow \quad \$4{,}000 = (\$K \times .952153) + ((\$K/2) \times \tfrac{2}{9}) + \$K/2 - ((\$K/2) \times .952153)$$

$$\Rightarrow \quad \$K = \$3{,}679$$

Answer is B.

## Question 29

The present value of the normal form of benefit can be set equal to the present value of the joint and survivor annuity.

$$(\$500 \times 12) \times (\ddot{a}^{(12)}_{\overline{5}|} + {}_{5|}\ddot{a}^{(12)}_{65}) = (\$X \times 12) \times (\ddot{a}^{(12)}_{65} - \tfrac{1}{2}(\ddot{a}^{(12)}_{65} - \ddot{a}^{(12)}_{65:65}))$$

$$(\$500 \times 12) \times (\ddot{a}^{(12)}_{\overline{5}|} + {}_{5}p_{65}v^{5}\ddot{a}^{(12)}_{70}) = (\$X \times 12) \times (\ddot{a}^{(12)}_{65} - \tfrac{1}{2}(\ddot{a}^{(12)}_{65} - \ddot{a}^{(12)}_{65:65}))$$

Solving for \$X: $\$X = 468$

Answer is C.