

SOCIETY OF ACTUARIES

AMERICAN SOCIETY OF PENSION ACTUARIES

JOINT BOARD FOR THE ENROLLMENT OF ACTUARIES

ENROLLED ACTUARIES BASIC EXAMINATION

MAY 2001 EA-1 EXAMINATION

2001

Data for Question 1 (4 points)

Selected values: $1000d^{(m)} = 85.256$

$1000d^{(2m)} = 85.715$

Question 1

In what range is $1000 i^{(3m)}$?

- [A] Less than 86.000
- [B] 86.000 but less than 86.400
- [C] 86.400 but less than 86.800
- [D] 86.800 but less than 87.200
- [E] 87.200 or more

2001

Data for Question 2 (2 points)

A loan of \$1,800 is to be repaid by a single payment of \$2,420.80 two years after the date of the loan. The terms of the loan are quoted using a nominal annual interest rate of 15%.

Question 2

What is the frequency of compounding?

- [A] monthly
- [B] every two months
- [C] quarterly
- [D] semiannually
- [E] annually

2001

Data for Question 3 (4 points)

Amount of a loan: \$100,000.

Number of originally scheduled level annual repayments: 30.

Time of first repayment: One year from the date of the loan.

Additional payments made with the 5th and 10th scheduled repayments: \$5,000 each.

Effective annual interest rate: 6%.

Subsequent to the two additional payments, the loan continues to be repaid by annual repayments of the original size, plus a smaller final repayment one year after the last full repayment.

Question 3

In what range is the total amount of interest saved due to the two additional payments?

- [A] Less than \$23,500
- [B] \$23,500 but less than \$23,600
- [C] \$23,600 but less than \$23,700
- [D] \$23,700 but less than \$23,800
- [E] \$23,800 or more

2001

Data for Question 4 (4 points)

Date of a loan: 1/1/2001.

Amount of loan: \$100,000.

Frequency of repayments: Quarterly.

Date of first repayment: 3/31/2001.

Number of repayments: 120.

Amount of each of the first 110 repayments: \$3,100.

Amount of last 10 repayments: Initial repayment of \$X, then doubling every quarter thereafter.

Interest rate: 12% per year, compounded quarterly.

Question 4

In what range is the amount of the final repayment?

- [A] Less than \$6,000
- [B] \$6,000 but less than \$12,000
- [C] \$12,000 but less than \$18,000
- [D] \$18,000 but less than \$24,000
- [E] \$24,000 or more

Data for Question 5 (5 points)

On 1/1/2002, Smith contributes \$2,000 into a new savings account that earns 5% interest, compounded annually. On each January 1 thereafter, he makes another deposit that is 97% of the prior deposit. This continues until he has made 20 deposits in all.

On each January 1 beginning on 1/1/2025, Smith makes annual withdrawals. There is to be a total of 25 withdrawals, with each withdrawal 4% more than the prior withdrawal, and the 25th withdrawal exactly depletes the account.

Question 5

In what range is the sum of the withdrawals made on 1/1/2025 and 1/1/2026?

- [A] Less than \$5,410
- [B] \$5,410 but less than \$5,560
- [C] \$5,560 but less than \$5,710
- [D] \$5,710 but less than \$5,860
- [E] \$5,860 or more

2001

Data for Question 6 (5 points)

Amount of a loan: \$25,000.

Term of loan: 8 Years.

Loan repayments: Quarterly, at the end of each quarter.

Interest rate: 8% per year, compounded semiannually.

The 11th and 12th scheduled repayments are not made.

The loan is renegotiated immediately after the due date of the 12th (2nd missed) scheduled repayment with the following provisions:

13th (1st renegotiated) scheduled repayment: \$X.

14th through 32nd repayments:

Each even-numbered repayment is \$200 greater than the immediately preceding (odd-numbered) repayment.

Each odd-numbered repayment is equal to the immediately preceding even-numbered repayment.

The loan is to be completely repaid over the original term.

Question 6

In what range is X?

- [A] Less than \$250
- [B] \$250 but less than \$255
- [C] \$255 but less than \$260
- [D] \$260 but less than \$265
- [E] \$265 or more

Data for Question 7 (3 points)

Repayment schedule for a loan:

End of Each Odd Numbered Year	Amount of Repayment
1	\$100
3	\$300
5	\$500
.	.
.	.
X	\$100X
.	.
.	.
25	\$2500

Interest rate: 6% per year, compounded annually.

A is the total of the payments to be made after the 15th year.

B is the present value of the remaining payments as of the beginning of the 16th year.

Question 7

In what range is A minus B?

- [A] Less than \$3,120
- [B] \$3,120 but less than \$3,150
- [C] \$3,150 but less than \$3,180
- [D] \$3,180 but less than \$3,210
- [E] \$3,210 or more

2001

Data for Question 8 (3 points)

Date of a loan: 1/1/2001.

Date of first repayment: 12/31/2001.

Frequency of repayments: Annually.

Term of loan: 4 years.

Amount of each repayment: \$1,000.

$$v = \frac{1}{1+i}$$

The sum of the principal repayments in years one and two is equal to $10v^2$ times the sum of the interest repayments in years three and four.

Question 8

In what range is v ?

- [A] Less than 0.930
- [B] 0.930 but less than 0.935
- [C] 0.935 but less than 0.940
- [D] 0.940 but less than 0.945
- [E] 0.945 or more

2001

Data for Question 9 (2 points)

Amount of a loan: \$1,000.

Date of loan: 1/1/2001.

Term of loan: 30 years

Date of first repayment: 1/1/2004.

Frequency of repayments: Every 3 years.

Interest rate: 4% per year, compounded annually.

Question 9

In what range is the principal repaid in the fifth repayment?

- [A] Less than \$85
- [B] \$85 but less than \$91
- [C] \$91 but less than \$97
- [D] \$97 but less than \$103
- [E] \$103 or more

2001

Data for Question 10 (3 points)

Issue date of a bond: January 1, 1994.

Term of bond: 15 years.

Par value of bond: \$10,000.

Coupons: 8% per annum, paid on June 30 and December 31.

Amortized value on July 1, 2001: \$13,741.11.

Amortized value on January 1, 2002: \$13,629.67.

Question 10

In what range is the redemption amount to be paid upon maturity?

- [A] Less than \$11,680
- [B] \$11,680 but less than \$11,750
- [C] \$11,750 but less than \$11,820
- [D] \$11,820 but less than \$11,890
- [E] \$11,890 or more

2001

Data for Question 11 (2 points)

Issue date of a bond: January 1, 2001.

Coupon dates: December 31, 2002 and every two years thereafter, with the final payment on December 31, 2010.

Coupon amount: \$60 each.

Investor's yield: 8% per annum.

Price of the bond at issue: \$691.49.

Amortized value on January 1, 2005: \$A.

Amortized value on January 1, 2007: \$B.

Question 11

In what range is the absolute value of $(\$A - \$B)$?

- [A] Less than \$63
- [B] \$63 but less than \$66
- [C] \$66 but less than \$69
- [D] \$69 but less than \$72
- [E] \$72 or more

Data for Question 12 (4 points)

A \$200,000, 30-year variable rate mortgage loan is obtained. The first monthly payment is due one month from the date of the loan. At the time the loan is obtained, the interest rate is 7.0%, compounded monthly.

On the second anniversary of the loan, the interest rate is increased to 7.5%, compounded monthly.

On the fourth anniversary of the loan, the interest rate is increased to 8.0%, compounded monthly, and remains fixed for the remainder of the mortgage repayment period.

Question 12

In what range is the total interest paid on the loan?

- [A] Less than \$310,000
- [B] \$310,000 but less than \$314,000
- [C] \$314,000 but less than \$318,000
- [D] \$318,000 but less than \$322,000
- [E] \$322,000 or more

2001

Data for Question 13 (2 points)

Purchase date of a perpetuity: 1/1/2001.

Date of first payment: 12/31/2001.

Frequency of payments: Annual.

Amount of each payment: \$1.00.

Interest rate: 6% per year, compounded annually.

Question 13

In what range is the absolute value of the difference between the modified duration of the perpetuity and the present value of the perpetuity?

- [A] Less than 0.20
- [B] 0.20 but less than 0.40
- [C] 0.40 but less than 0.60
- [D] 0.60 but less than 0.80
- [E] 0.80 or more

2001

Data for Question 14 (5 points)

Consider the following 3 portfolios:

Portfolio A: 4-year bonds with 7% annual coupons.
5-year zero-coupon bonds.

Portfolio B: 3-year bonds with 7% annual coupons.
5-year zero-coupon bonds.

Portfolio C: 4-year zero-coupon bond with a maturity value of \$10,000.

All bonds yield 7%.

The amount of each type of bond within a given portfolio is selected such that all three portfolios have the same present value and the same modified duration.

$\$X$ = the amount invested in 5-year zero-coupon bonds in Portfolio A.

$\$Y$ = the amount invested in 5-year zero-coupon bonds in Portfolio B.

Question 14

In what range is the absolute value of $(\$X - \$Y)$?

- [A] Less than \$2,000
- [B] \$2,000 but less than \$2,300
- [C] \$2,300 but less than \$2,600
- [D] \$2,600 but less than \$2,900
- [E] \$2,900 or more

2001

Data for Question 15 (2 points)

A 12-year annual annuity has its first payment of \$10,000 due one year from purchase. Subsequent payments will be indexed to the excess of the percentage increase in the Consumer Price Index (CPI) over 3%.

Interest rate: 8% per year, compounded annually.

\$X = the present value of the annuity if the constant rate of increase in the CPI is 6%.

\$Y = the present value of the annuity if the constant rate of increase in the CPI is 4%.

Question 15

In what range is the absolute value of (\$X minus \$Y)?

- [A] Less than \$7,745
- [B] \$7,745 but less than \$7,925
- [C] \$7,925 but less than \$8,105
- [D] \$8,105 but less than \$8,285
- [E] \$8,285 or more

2001

Data for Question 16 (4 points)

Deaths are uniformly distributed over $[0,100]$. The interest rate is 6% compounded annually.

Question 16

In what range is $\$100a_{60}$?

- [A] Less than \$1,000
- [B] \$1,000 but less than \$1,005
- [C] \$1,005 but less than \$1,010
- [D] \$1,010 but less than \$1,015
- [E] \$1,015 or more

2001

Data for Question 17 (3 points)

Interest rate: 7% per year, compounded annually.

$$l_{105+t} = (950)(1.07)^{-t}, 0 \leq t \leq 5$$

Smith is currently age 105. If she survives until age 106, she will become entitled to a two-year certain and life annuity that pays \$1,000 at the end of each year.

\$X is the present value at Smith's current age of this annuity.

Question 17

In what range is \$X?

- [A] Less than \$1,490
- [B] \$1,490 but less than \$1,600
- [C] \$1,600 but less than \$1,710
- [D] \$1,710 but less than \$1,820
- [E] \$1,820 or more

2001

Data for Question 18 (3 points)

Current age of a mortgagee: 57.

Frequency of level mortgage payments: Annual, at the end of each year.

Remaining mortgage amount: \$50,000.

Remaining mortgage term: 3 years.

Interest rate: 5% per year, compounded annually.

The mortgage is insured by the purchase of a 3-year decreasing term insurance policy with a death benefit equal to the mortgage balance at the end of the year of death.

Selected values:

$\frac{x}{56}$	$\frac{l_x}{9604}$
57	9574
58	9541
59	9505
60	9467
61	9424

Question 18

In what range is the current present value of the insurance benefit?

- [A] Less than \$350
- [B] \$350 but less than \$360
- [C] \$360 but less than \$370
- [D] \$370 but less than \$380
- [E] \$380 or more

2001

Data for Question 19 (3 points)

Consider the following:

Current age of worker: 55.

Age at first payment: 65.

Annual lifetime retirement income: \$50,000 paid once each year at the beginning of the year for life.

Selected values:

$${}_{10}p_{55} = 0.92$$

$${}_{20}p_{55} = 0.624$$

$$a_{65} = 8.897$$

$$a_{75} = 6.217$$

$$i = 0.06$$

A provision is added that upon retirement at age 65 the first ten payments are guaranteed.

Question 19

In what range is the additional cost, at age 55, of this new provision?

- [A] Less than \$17,000
- [B] \$17,000 but less than \$18,200
- [C] \$18,200 but less than \$19,400
- [D] \$19,400 but less than \$20,600
- [E] \$20,600 or more

2001

Data for Question 20 (3 points)

Selected values:

$$l_{50} = 100,000$$

$$l_{51} = 98,000$$

$$l_{52} = 95,550$$

Additional information:

Uniform distribution of deaths is assumed from age 50 to 51.

$\overset{\circ}{e}_{52}$ equals 26 years.

$${}_t p_{51} = (p_{51})^t, \quad 0 < t \leq 1$$

Question 20

In what range is $\overset{\circ}{e}_{50}$?

- [A] Less than 26.45 years
- [B] 26.45 years but less than 26.65 years
- [C] 26.65 years but less than 26.85 years
- [D] 26.85 years but less than 27.05 years
- [E] 27.05 years or more

2001

Data for Question 21 (3 points)

The following facts relate to a stationary population:

Number of lives attaining age 20 each year: 1,080

Number of persons living at age 20 and older: 21,600

Number of persons living at age 50 and older: 2,700

Average age at death of those dying between ages 20 and 50: $33 \frac{1}{3}$

Question 21

In what range is $\overset{\circ}{e}_{50}$?

- [A] Less than 9.5 years
- [B] 9.5 years but less than 9.8 years
- [C] 9.8 years but less than 10.1 years
- [D] 10.1 years but less than 10.4 years
- [E] 10.4 years or more

2001

Data for Question 22 (3 points)

Age of Smith on January 1, 2001: 40

Age of Brown on January 1, 2001: 41

Selected values:

$$e_{40} = 16.5 \text{ years}$$

$$e_{41} = 16.2 \text{ years}$$

$$e_{42} = 16.0 \text{ years}$$

$$e_{43} = 15.8 \text{ years}$$

Question 22

In what range is the probability that one of Smith and Brown will die in 2001 and the other in 2002?

- [A] Less than 0.00360
- [B] 0.00360 but less than 0.00380
- [C] 0.00380 but less than 0.00400
- [D] 0.00400 but less than 0.00420
- [E] 0.00420 or more

2001

Data for Question 23 (5 points)

Consider the following:

Smith, age 20, with ${}_n p_{20} = (0.95)^n$, $n \geq 0$

Brown, age 25, with ${}_n p_{25} = (0.90)^n$, $n \geq 0$

Green, age 30, with ${}_n p_{30} = (0.85)^n$, $n \geq 0$

Question 23

In what range is the probability that all three are alive five years from now and at least two are alive 15 years from now?

- [A] Less than 0.061
- [B] 0.061 but less than 0.070
- [C] 0.070 but less than 0.079
- [D] 0.079 but less than 0.088
- [E] 0.088 or more

2001

Data for Question 24 (4 points)

An annuity of \$10,000 is payable at the end of each year to the annuitant while both the annuitant and his spouse are alive.

The annuity is also paid to the annuitant, if alive, for 10 years after his spouse's death.

However, in no event will payments be made after 20 years from the present time.

Current data: Annuitant's age, 65
Spouse's age, 60

$$a_{65:\overline{10}|} = 7.72174$$

$$v^{10} {}_{10}p_{65} = 0.54544$$

$$a_{75:60:\overline{10}|} = 6.49715$$

$$a_{75:70:\overline{10}|} = 6.17348$$

$$v^{10} {}_{10}p_{65:60} = 0.50735$$

Question 24

In what range is the present value of the annuity?

- [A] Less than \$108,500
- [B] \$108,500 but less than \$110,000
- [C] \$110,000 but less than \$111,500
- [D] \$111,500 but less than \$113,000
- [E] \$113,000 or more

Data for Question 25 (5 points)

Consider the following:

- (i) The probability that 3 persons aged 30, 40, and 50 will all live at least 10 years is 0.758.
- (ii) The probability that a person aged 55 will die within 5 years, while a person aged 50 will be alive at the end of 5 years, is 0.063.
- (iii) The probability that 4 persons aged 30, 35, 40, and 45 will all live for at least 5 years, while a 5th person aged 50 will not be alive at the end of 5 years, is 0.045.

Question 25

In what range is the probability that a person aged 30 will be alive at the end of 25 years?

- [A] Less than 0.800
- [B] 0.800 but less than 0.825
- [C] 0.825 but less than 0.850
- [D] 0.850 but less than 0.875
- [E] 0.875 or more

2001

Data for Question 26 (3 points)

A sample of 100 lives is to be observed from age 50 to 51.

Selected information:

16 deaths are expected to occur.

Both deaths and withdrawals are uniformly distributed in their respective associated single decrement tables.

$$q_{50}^{(w)} = 0.4$$

Question 26

In what range is $q_{50}^{(w)}$?

- [A] Less than 0.365
- [B] 0.365 but less than 0.370
- [C] 0.370 but less than 0.375
- [D] 0.375 but less than 0.380
- [E] 0.380 or more

Data for Question 27 (3 points)

For a two-decrement service table, the associated two single-decrement service tables are:

<u>Decrement #1</u>		<u>Decrement #2</u>	
\underline{x}	\underline{l}_x	\underline{x}	\underline{l}_x
40	100	40	100
41	90	41	70

Each decrement has a uniform distribution within the two-decrement service table.

Question 27

In what range is $q_{40}^{(2)}$?

- [A] Less than 0.28500
- [B] 0.28500 but less than 0.28520
- [C] 0.28520 but less than 0.28540
- [D] 0.28540 but less than 0.28560
- [E] 0.28560 or more

2001

Data for Question 28 (5 points)

At retirement, a pensioner and spouse can elect any one of the following four actuarially equivalent forms of annuity payments:

- I) \$4,000 per month for the pensioner's lifetime.
- II) \$3,600 per month for the pensioner's lifetime, and \$1,800 per month for the lifetime of the surviving spouse upon the death of the pensioner.
- III) \$3,582 per month for the joint lifetime of the pensioner and spouse, \$1,791 per month for the remaining lifetime of the surviving spouse if the pensioner dies first, and \$4,000 per month for the remaining lifetime of the pensioner if the spouse dies first.
- IV) \$K per month for the joint lifetime of the pensioner and spouse, and $\$K/2$ per month for the remaining lifetime of the survivor after the first death.

Question 28

In what range is \$K?

- [A] Less than \$3,660
- [B] \$3,660 but less than \$3,690
- [C] \$3,690 but less than \$3,720
- [D] \$3,720 but less than \$3,750
- [E] \$3,750 or more

2001

Data for Question 29 (3 points)

A 5-year certain and life annuity issued to Brown on 1/1/2001 provides a monthly income of \$500 beginning 1/1/2001.

The annuity is actuarially equivalent to a joint and survivor annuity that pays the following amounts to Brown and his beneficiary, both age 65 on 1/1/2001:

1. \$X per month beginning 1/1/2001 and payable as long as Brown is alive, whether or not his beneficiary is alive.
2. \$X/2 beginning upon Brown's death, payable monthly for the lifetime of Brown's beneficiary.

Selected values:

$$i = 0.05$$

$${}_5p_{65} = 0.95609$$

$$\ddot{a}_{65:65}^{(12)} = 10.87$$

$$\ddot{a}_{65}^{(12)} = 12.80$$

$$\ddot{a}_{70}^{(12)} = 11.27$$

Question 29

In what range is \$X?

- [A] Less than \$400
- [B] \$400 but less than \$450
- [C] \$450 but less than \$500
- [D] \$500 but less than \$550
- [E] \$550 or more

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ANSWER KEY

MAY 2001 EA-1

1. C
2. B
3. B
4. B
5. D

6. C
7. C
8. C
9. B
10. C

11. E
12. D
13. A
14. B
15. B

16. B
17. B
18. B
19. A
20. C

21. C
22. C
23. B
24. D
25. B

26. A
27. E
28. B
29. C