ASM Study Manual for Exam P, Three Practice Exams, First Edition (May 2009)
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Errata

Posted December 21, 2009
In Problem 24, Practice Examination 23, the first sentence should read:

The moment-generating function of a random variable X is $M_X(t) = \frac{4}{(2-t)^3} + \frac{e^{e^t}}{2e}$, for $t < 2$.

Posted December 21, 2009
In Problem 18, Practice Examination 23, answer choice D should be:

$$\frac{1}{2} - \binom{n}{\frac{n}{2}} \left(\frac{1}{2}\right)^{n+1}.$$

Posted December 20, 2009
Problem 4 in Practice Examination 23 should be:
You are given two random variables X and Y such that $E(X) = 0$, $E(Y) = -1$, $\mathrm{Var}(X) = 1$, $\mathrm{Var}(Y) = 4$, and $\mathrm{Var}(X+Y) = 7$. Find a value of a parameter a such that $X + Y$ and $X + aY$ are uncorrelated.

A. $-\frac{2}{5}$ B. $-\frac{1}{3}$ C. 0 D. $\frac{1}{3}$ E. $\frac{1}{6}$

Solution.
We have

$$9 = \mathrm{Var}(X+Y) = \mathrm{Var}(X) + \mathrm{Var}(Y) + 2\mathrm{Cov}(X,Y) = 1 + 4 + 2\mathrm{Cov}(X,Y)$$

so that

$$\mathrm{Cov}(X,Y) = 1.$$

Also

$$\begin{aligned}\mathrm{Cov}(X+Y, X+aY) &= \mathrm{Cov}(X,X) + \mathrm{Cov}(X,aY) + \mathrm{Cov}(Y,X) + \mathrm{Cov}(Y,aY) = \\ &= \mathrm{Var}(X) + a\mathrm{Cov}(X,Y) + \mathrm{Cov}(X,Y) + a\mathrm{Var}(Y) = \\ &= 1 + a + 1 + 4a = 2 + 5a.\end{aligned}$$

Now recall that $X + Y$ and $X + aY$ are uncorrelated if and only if their covariance is zero. This means that $a = -\frac{2}{5}$.
Answer A.

Posted December 20, 2009
Problem 19 in Practice Examination 22 was meant to ask for $\Pr(Y - X > 1)$, **not** $\Pr(Y - X < 1)$.

Posted December 4, 2009
Problem 18 in Practice Examination 22 should be:
Dr. Ostaszewski's online exercise posted December 12, 2009
Let X_1, X_2 be a random sample from a Poisson distribution with mean 1 and let $Y_{(1)}$ be its first order statistic. Find $\Pr\left(Y_{(1)} = 1\right)$.

A. $\dfrac{2e-3}{e^2}$ B. $3e^{-1} - 2e^{-2}$ C. $\dfrac{e-1}{e}$ D. $\dfrac{2e-1}{e^2}$ E. $\dfrac{1}{e}$

Solution.
We have

$$\begin{aligned}\Pr\left(Y_{(1)} = 1\right) &= \Pr\left(\left(\{X_1 = 1\} \cap \{X_2 > 1\}\right) \cup \left(\{X_1 > 1\} \cap \{X_2 = 1\}\right) \cup \left(\{X_1 = 1\} \cap \{X_2 = 1\}\right)\right) = \\ &= \Pr(X_1 = 1) \cdot \left(1 - \Pr(X_2 = 1) - \Pr(X_2 = 0)\right) + \\ &\quad + \left(1 - \Pr(X_1 = 1) - \Pr(X_1 = 0)\right) \cdot \Pr(X_2 = 1) + \Pr(X_1 = 1) \cdot \Pr(X_2 = 1).\end{aligned}$$

Now we have

$$\Pr(X_1 = 1) \cdot \left(1 - \Pr(X_2 = 1) - \Pr(X_2 = 0)\right) = e^{-1} \cdot \frac{1^1}{1!} \cdot \left(1 - e^{-1} \cdot \frac{1^1}{1!} - e^{-1} \cdot \frac{1^0}{0!}\right) = e^{-1} - 2e^{-2},$$

$$\left(1 - \Pr(X_1 = 1) - \Pr(X_1 = 0)\right) \cdot \Pr(X_2 = 1) = \left(1 - e^{-1} \cdot \frac{1^1}{1!} - e^{-1} \cdot \frac{1^0}{0!}\right) \cdot e^{-1} \cdot \frac{1^1}{1!} = e^{-1} - 2e^{-2},$$

and

$$\Pr(X_1 = 1) \cdot \Pr(X_2 = 1) = e^{-1} \cdot \frac{1^1}{1!} \cdot e^{-1} \cdot \frac{1^1}{1!} = e^{-2}.$$

This results in

$$\Pr\left(Y_{(1)} = 1\right) = e^{-1} - 2e^{-2} + e^{-1} - 2e^{-2} + e^{-2} = 2e^{-1} - 3e^{-2} = \frac{2}{e} - \frac{3}{e^2} = \frac{2e-3}{e^2}.$$

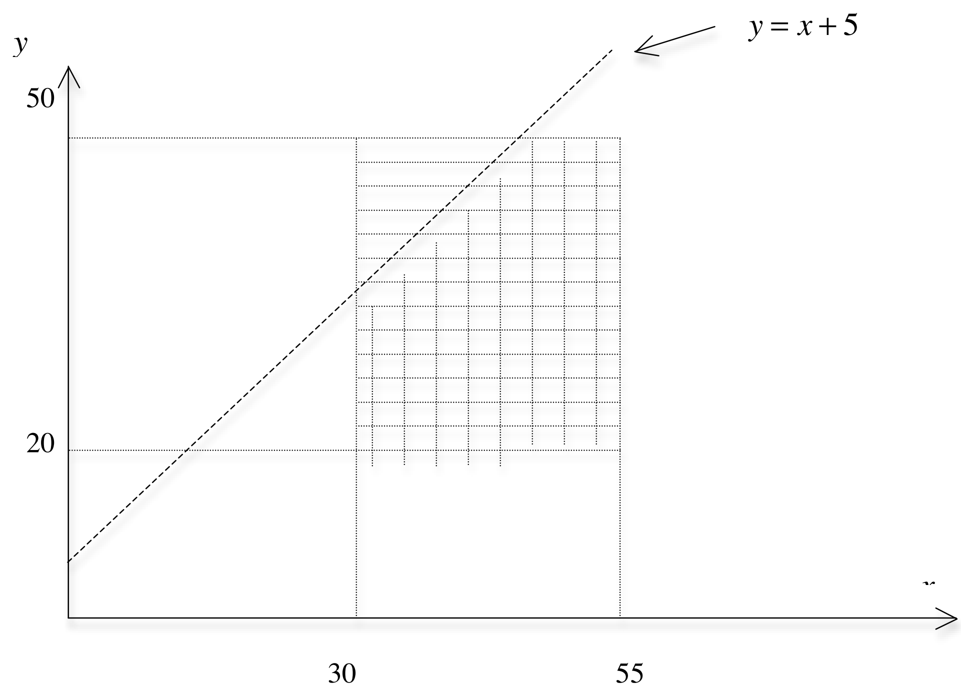
Answer A.

Posted November 7, 2009
Problem 2 in Practice Examination 23 should be:
Dr. Ostaszewski's online exercise posted November 7, 2009
Widgets is an office supply store that accepts packages for delivery by another company, Very Reliable Shipping (VRS). A VRS representative arrives at a Widgets store at a time uniformly distributed between 6:30 p.m. and 6:55 p.m. Packages are brought in from the

place where they are received from customers using Widgets as a shipping outlet by an employee who arrives at a time uniformly distributed between 6:20 p.m. and 6:50 p.m. The employee puts packages in an area from which the VRS representative picks them up. The VRS representative waits five minutes after arrival, and then takes all packages available and leaves. The arrival times of the VRS representative and of the Widgets employee are independent random variables. If a customer brings a package to Widgets in the morning, what is the probability that the package will be received by VRS the same day?

A. 0.42 B. 0.50 C. 0.58 D. 0.67 E. 0.85

Solution.
Let *X* be the number of minutes since 6 p.m. at the time when the VRS rep shows up, and let *Y* be the number of minutes since 6 p.m. at the time when the Widgets employees brings in the packages. If $Y < X + 5$, the VRS rep will take any packages left from the previous day and today's packages, otherwise the VRS rep will only take packages from the previous day. Therefore, we need to find $\Pr(Y < X + 5)$. But *X* is uniform on the interval $[30,55]$ while *Y* is uniform on the interval $[20,50]$, with the two being independent, so that the joint distribution of *X* and *Y* is uniform on $[30,55]\times[20,50]$, as shown in the figure below, indicated with horizontal lines.



The vertical lines show the region where $Y < X + 5$. The area of that region equals the difference between the area where the density is positive, $25 \cdot 30 = 750$, and the area of the small cut-out triangle in the left-upper corner, which is $\frac{1}{2} \cdot 15 \cdot 15 = 112.50$, so that the area sought is $750 - 112.50 = 637.50$. Since the joint distribution is uniform, the probability sought is just the ratio of areas:

$$\frac{\text{Area of the region where } Y < X + 5}{\text{Area of the rectangle where joint density is positive}} = \frac{637.50}{750} = 0.85.$$

Answer E.

Posted May 20, 2009
The last formula in the solution Problem 23, Practice Examination 21, should be:

$$\Pr(X = 4) = \Pr(X = 4 | \text{Heads}) \cdot \Pr(\text{Heads}) + \Pr(X = 4 | \text{Tails}) \cdot \Pr(\text{Tails}) =$$

$$= \frac{2}{9} \cdot \frac{1}{2} + \left(\underbrace{\frac{1}{9} \cdot \frac{1}{9}}_{1+3} + \underbrace{\frac{2}{9} \cdot \frac{2}{9}}_{2+2} + \underbrace{\frac{1}{9} \cdot \frac{1}{9}}_{3+1} \right) \cdot \frac{1}{2} = \frac{1}{9} + \frac{3}{81} = \frac{3}{27} + \frac{1}{27} = \frac{4}{27}.$$

instead of

$$\Pr(X = 4) = \Pr(X = 4 | \text{Heads}) \cdot \Pr(\text{Heads}) + \Pr(X = 4 | \text{Tails}) \cdot \Pr(\text{Heads}) =$$

$$= \frac{2}{9} \cdot \frac{1}{2} + \left(\underbrace{\frac{1}{9} \cdot \frac{1}{9}}_{1+3} + \underbrace{\frac{2}{9} \cdot \frac{2}{9}}_{2+2} + \underbrace{\frac{1}{9} \cdot \frac{1}{9}}_{3+1} \right) \cdot \frac{1}{2} = \frac{1}{9} + \frac{3}{81} = \frac{3}{27} + \frac{1}{27} = \frac{4}{27}.$$

that is, the word Heads **at the end of the first line of the formula was a typo, it should be** Tails.

Posted May 20, 2009
The second sentence of Problem 2 in Practice Examination 22 should be:
If the driver is “bad“ driver, he is from a Poisson population with a mean of 5 claim per year.
instead of
If the driver is “bad“ driver, he is from a Poisson population with a mean of 8 claim per year.
In other words, there is a typo, 8 **is written instead of** 5.

Posted May 20, 2009
Answer choices in Problem 29, Practice Examination 23, should be:
A. 0.0175 B. 0.04000 C. 0.2684 D. 0.6242 E.0.9596
Answer D was mistyped as D. 0.9596.