



*Three formulas for  ${}_{t|u}q_x$  in terms of  
non-deferred  $p$ 's and  $q$ 's*



$${}_{t|u}q_x = {}_t p_x {}_u q_{x+t}$$

$${}_{t|u}q_x = {}_t p_x - {}_{t+u} p_x$$

$${}_{t|u}q_x = {}_{t+u} q_x - {}_t q_x$$



*$k p_x$  in terms of  $l$ 's*

## Survival Distributions



$${}_k p_x = \frac{l_{x+k}}{l_x}$$



*$kq_x$  in terms of  $l$ 's and  $d$ 's*



$${}_kq_x = \frac{{}_k d_x}{l_x} = \frac{l_x - l_{x+k}}{l_x}$$



*$t|uq_x$  in terms of  $l$ 's and  $d$ 's*

$${}_{t|u}q_x = \frac{{}_u d_{x+t}}{l_x} = \frac{l_{x+t} - l_{x+t+u}}{l_x}$$





*Definition of  ${}_tq_x$  in terms of probabilities of  $X$ ,  
the random variable for age at death.*



$${}_tq_x = \Pr(x < X \leq x + t \mid X > x)$$



${}_s q_x$  under UDD,  $s \leq 1$



$${}_s q_x = {}_s q_x$$



${}_s p_x$  under UDD,  $s \leq 1$



$${}_s p_x = 1 - {}_s q_x$$



$l_{x+s}$  *under UDD*,  $s \leq 1$



$$l_{x+s} = l_x - s d_x$$





${}_{1-s}q_{x+s}$  *under UDD*,  $s \leq 1$



$${}_{1-s}q_{x+s} = \frac{(1-s)q_x}{1-sq_x}$$



${}_s q_{x+t}$  under UDD,  $s + t \leq 1$



$${}_s q_{x+t} = \frac{{}_s q_x}{1 - {}_t q_x}$$



$\mu_{x+s}$  *under UDD*,  $s \leq 1$



$$\mu_{x+s} = \frac{q_x}{1 - sq_x}$$



${}_s p_x \mu_{x+s}$  under UDD,  $s \leq 1$



$${}_s p_x \mu_{x+s} = q_x$$





*Recursive formula for UL account value if  
corridor of  $\gamma$  applies*

$$AV_t = \frac{(AV_{t-1} + P_t - e_t)(1 + i)}{1 + v_q q_{x+t-1}(1 + i)(\gamma - 1)} \quad (67.10)$$

$$AV_t = \frac{(AV_{t-1} + P_t - e_t)(1 + i)}{1 + q_{x+t-1}(\gamma - 1)} \text{ if } i_q = i \quad (67.9)$$



*Recursive formula for UL cost of insurance if  
corridor of  $\gamma$  applies*

$$\text{ColI}_t = \frac{v_q q_{x+t-1}(\gamma - 1)(AV_{t-1} + P_t - e_t)(1 + i)}{1 + v_q q_{x+t-1}(1 + i)(\gamma - 1)}$$



*Definition of profit in UL profit test*

## Profit Tests



$$AV_{t-1} + G_t - e_t + I_t - EDB_t - ESB_t - EAV_t$$

where

- $e_t$  is annual expense.
- $EDB$  is expected total death benefits including account value settlement expense.
- $ESB$  is expected total surrender benefits including account value and settlement expense.
- $EAV = AV_t \left(1 - q_{x+t-1}^{(d)}\right) \left(1 - q_{x+t-1}^{(w)}\right)$ .



*Gross premium reserve recursion with  
surrenders*



$${}_kV = \frac{({}_{k-1}V + G_{k-1} - e_{k-1})(1 + i) - q_{x+k-1}^{(d)}(b_k + E_k^{(d)}) - q_{x+k-1}^{(w)}(CV_k + E_k^{(w)})}{p_{x+k-1}^{(\tau)}}$$