



Definition of bias



$$\text{bias}_{\hat{\theta}}(\theta) = \mathbf{E}[\hat{\theta}] - \theta$$



Bias of sample mean

Estimator Quality



0



Bias of biased sample variance



$$\text{bias}_{\hat{\sigma}^2}(\sigma^2) = -\frac{\sigma^2}{n}$$



Definition of consistency



***Consistency** means that the probability that the estimator is different from the parameter by more than ϵ goes to 0 as the sample size goes to infinity.*



Sufficient condition for consistency

Estimator Quality



Estimator is asymptotically unbiased and its variance goes to 0 as the sample size goes to infinity.



*Definition of relative efficiency of estimator θ_1
to estimator θ_2*

Estimator Quality



$$\frac{\text{Var}(\hat{\theta}_2)}{\text{Var}(\hat{\theta}_1)}$$



Definition of mean square error of estimator



$$\text{MSE}_{\hat{\theta}}(\theta) = \mathbf{E}[(\hat{\theta} - \theta)^2]$$



Formula for mean square error



$$\text{MSE}_{\hat{\theta}}(\theta) = \text{bias}_{\hat{\theta}}(\theta)^2 + \text{Var}(\hat{\theta})$$



Definition of UMVUE



A uniformly minimum variance unbiased estimator is an unbiased estimator has the lowest variance of any unbiased estimator regardless of the true value of θ , the estimated parameter.



Definition of exponential family



$$f(y; \theta) = \exp(a(y)b(\theta) + c(\theta) + d(y))$$



*Canonical form of exponential family and
natural parameter*



Canonical form: $a(y) = y$
Natural parameter: $b(\theta)$



Examples of members of exponential family



Extended Linear Model

- *binomial*
- *normal*
- *Poisson*
- *exponential*
- *gamma*
- *inverse Gaussian*
- *negative binomial*
- *compound Poisson/gamma*



$\mathbf{E}[Y]$ for Y exponential in canonical form



$$\mathbf{E}[Y] = -\frac{c'(\theta)}{b'(\theta)}$$



$\text{Var}(Y)$ for Y exponential in canonical form



$$\text{Var}(Y) = \frac{b''(\theta)c'(\theta) - c''(\theta)b'(\theta)}{(b'(\theta))^3}$$



Definition of Tweedie distribution

Extended Linear Model



$$\text{Var}(Y) = a \mathbf{E}[Y]^p$$