

Errata for the ASM Study Manual for Exam P, Eleventh Edition
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Posted September 21, 2011

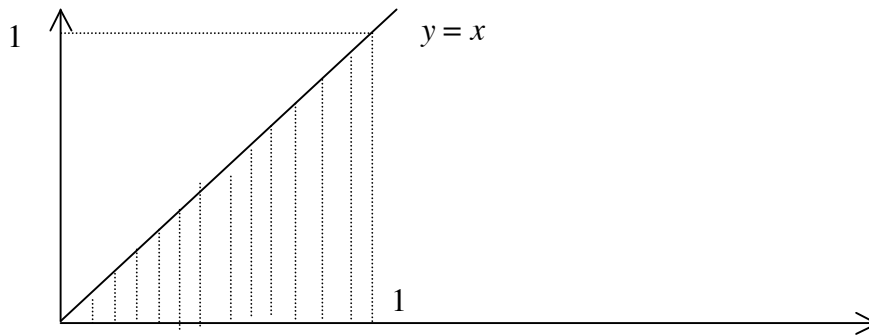
The solution of Problem 29 in Practice Examination 19 should be:

Solution.

The joint density, where positive, for $0 < x < 1$ and $0 < y < x$, is

$$f_{X,Y}(x,y) = f_Y(y|X=x) \cdot f_X(x) = \frac{1}{x} \cdot 2x = 2.$$

The region where that joint density is positive is indicated with dotted lines in the graph below



Since the joint density is uniform, the conditional distribution of X given that $Y = y$ is uniform on the range of values of x determined by the condition $0 < y < x < 1$, i.e., the interval $(y,1)$, so that the variance is $\frac{(1-y)^2}{12}$.

Answer E.

Posted August 31, 2011

The first sentence of the solution of Problem 14 in Practice Examination 18 should be:

Chi-square distribution is obtained as a sum of squares of independent identically distributed standard normal random variables, so we need to standardize these variables, add squares of those standardized variables, and hope we get one of the answers.

Posted August 6, 2011

In the third line of Problem 21 in Practice Examination 15, the words “and integer” should be “an integer”.

Posted August 6, 2011

In the solution of Problem 6 in Practice Examination 9, in the first line, the words “for and” should be “for any”.

Posted August 6, 2011

In the solution of Problem 1 in Practice Examination 9, the formula in the fifth line should be

$$\Pr(Y_{(1)} = 3) = \Pr(E - F) = \Pr(E) - \Pr(F).$$

instead of

$$\Pr(Y_{(1)} = 3) = \Pr(F) - \Pr(F).$$

Posted August 4, 2011

The solution of Problem 7 in Practice Examination 13 should be rephrased as follows:

The event of at least one color not being represented is the complement of the event of all three colors being represented, and all colors being represented simply means that we pick one red ball out of 3, one green ball out of 2, and one yellow ball out of 1. Thus

$$\begin{aligned} \Pr(\text{At least one color not drawn}) &= \\ &= 1 - \Pr(\text{All colors drawn}) = 1 - \frac{\binom{3}{1} \cdot \binom{2}{1} \cdot \binom{1}{1}}{\binom{6}{3}} = 0.70. \end{aligned}$$

You could also argue as follows. We are looking for the probability that all three balls are of the same color, or of two colors only. We have

$$\Pr(3 \text{ balls of one color}) = \Pr(3 \text{ red}) = \frac{1}{\binom{6}{3}} = \frac{3! \cdot 3!}{6!} = \frac{6}{4 \cdot 5 \cdot 6} = \frac{1}{20},$$

and

$$\begin{aligned} \Pr(3 \text{ balls of two colors only}) &= \Pr(2 \text{ red} + 1 \text{ green}) + \Pr(2 \text{ red} + 1 \text{ yellow}) + \\ &+ \Pr(2 \text{ green} + 1 \text{ red}) + \Pr(2 \text{ green} + 1 \text{ yellow}) = \\ &= \frac{\binom{3}{2} \cdot \binom{2}{1}}{\binom{6}{3}} + \frac{\binom{3}{2} \cdot \binom{1}{1}}{\binom{6}{3}} + \frac{\binom{2}{2} \cdot \binom{3}{1}}{\binom{6}{3}} + \frac{\binom{2}{2} \cdot \binom{1}{1}}{\binom{6}{3}} = \frac{13}{20}, \end{aligned}$$

so that the total probability is $\frac{1}{20} + \frac{13}{20} = \frac{7}{10}$.

Answer B.

Posted August 3, 2011

In Practice Examination 20, Problem 6, in the solutions part of the examination, answer choices were not listed. They are listed in the questions part of the examination.

Posted August 3, 2011

The last sentence of Problem 11 in Practice Examination 16 should be:

Calculate the variance of Y given that $X > 3$ and $Y > 3$.

instead of

Calculate the variance of Y given that and $X > 3$ and $Y > 3$.

Posted August 3, 2011

In Problem 7 in Practice Examination 14, the solution should start with the words

Because you studied this manual

instead of

Because you studied his manual

Posted July 30, 2011

Problem 7 in Practice Examination 19 should start with

Let X_1, X_2, \dots, X_{36} and Y_1, Y_2, \dots, Y_{49} be independent random samples from distributions ...

instead of

Let x_1, x_2, \dots, x_{36} and y_1, y_2, \dots, y_{49} be independent random samples from distributions ...

Posted July 30, 2011

The first line of the formula in the solution of Problem 6 in Practice Examination 19 should be

$$\Pr(2X - X^2 > 0) = \Pr(X(2 - X) > 0) = \Pr(X(X - 2) < 0) =$$

instead of

$$\Pr(2X - X^2 > 0) = \Pr(X(2 - X) > 0) = \Pr(X(X - 2) > 0) =$$

Posted July 28, 2011

In the formula in the solution of Problem 26 in Practice Examination 17, the formula should be

$$\begin{aligned} f_X(x) &= F'_X(x) = -\frac{d}{dx} \sum_{k=0}^3 \frac{x^k \cdot e^{-x}}{k!} = -\frac{d}{dx} \left(e^{-x} + xe^{-x} + \frac{1}{2}x^2 \cdot e^{-x} + \frac{1}{6}x^3 \cdot e^{-x} \right) = \\ &= -\left(-e^{-x} + (e^{-x} - xe^{-x}) + \left(xe^{-x} - \frac{1}{2}x^2 \cdot e^{-x} \right) + \left(\frac{1}{2}x^2 \cdot e^{-x} - \frac{1}{6}x^3 \cdot e^{-x} \right) \right) = \frac{1}{6}x^3 \cdot e^{-x}. \end{aligned}$$

The formula was missing a minus sign in the second line just after the first parenthesis. The rest of the solution is unaffected.

Posted July 28, 2011

In the solution of Problem 16 in Practice Examination 17, the word “bad” in the first sentence should be replaced by “bag”.

Posted July 28, 2011

The second sentence of the solution of Problem 12 in Practice Examination 16 should be:

Box 1 contains 1 blue and 4 red marbles, box 2 contains 2 blue and 3 red marbles and box 3 contains 3 blue and 2 red marbles.

instead of

Box 1 contains 1 blue and 4 red marbles, box 2 contains 2 blue and 3 red marbles and box 3 contains 3 red and 2 blue marbles.

The rest of the solution is unaffected by this typo.

Posted July 26, 2011

In Problem 9 in Practice Examination 14, the third condition should be:

(ii) The future lifetimes follow a Weibull distribution with $\alpha = 1.5$ and $\beta = 2.0$ for smokers, and $\alpha = 2.0$ and $\beta = 2.0$ for nonsmokers.

Also, the survival function of the Weibull distribution should be given as

$$s_T(t) = e^{-\left(\frac{t}{\alpha}\right)^\beta}.$$

Posted June 26, 2011

In Problem 29 in Practice Examination 19, the last sentence should be:

Find the variance of the conditional distribution of X , given $Y = y$.

Posted March 10, 2011

The second sentence of the solution of Problem 10 in Practice Examination 1 should be:

As the policy has a deductible of 1 (thousand), the claim payment is

$$Y = \begin{cases} 0, & \text{when there is no damage, with probability 0.94,} \\ \max(0, X - 1), & \text{when } 0 < X < 15, \text{ with probability 0.04,} \\ 14, & \text{in the case of total loss, with probability 0.02.} \end{cases}$$

Posted January 25, 2011

The last two sentences of the solution of Problem 11 in Practice Examination 16 should be replaced by

But the memoryless property of the exponential distribution tells us that Y and $(Y - 3|Y > 3)$ have the same distribution. Note, however, that

$$(Y|Y > 3) = 3 + (Y - 3|Y > 3),$$

so that

$$\text{Var}(Y|Y > 3) = \text{Var}(Y - 3|Y > 3) = \text{Var}(Y).$$

This implies that

$$\text{Var}(Y|\{X > 3\} \cap \{Y > 3\}) = \text{Var}(Y|Y > 3) = \text{Var}(Y) = \frac{1}{2^2} = 0.25.$$

Answer A.

Posted January 15, 2011

In Problem 16 in Practice Examination 6, the calculation of the second moment of X should be:

$$E(X^2) = \frac{1}{4} \cdot 0^2 + \frac{3}{4} \cdot \underbrace{(1+1)}_{\text{Second moment of } T} = \frac{3}{2}.$$

instead of

$$E(X^2) = E(X) = \frac{1}{4} \cdot 0^2 + \frac{3}{4} \cdot \underbrace{(1+1)}_{\text{Second moment of } T} = \frac{3}{2}.$$

Posted September 25, 2010

In Problem 22 in Practice Examination 19, answer choice C, the condition in the first line of the definition of $F_X(x)$ should be $x \leq 2$, not $x \leq 1$.

Posted August 2, 2010

In the solution of Problem 2 in Practice Examination 20, the sentence

Let X be the number of tails that are tossed that are tossed until the third head occurs.
should be

Let X be the number of tails that are tossed until the third head occurs.

Posted July 28, 2010

The first line of the first formula in the solution of Problem 13 in Practice Examination 20 has a typo in the denominator, it should say $>$ instead of $<$.

Posted July 24, 2010

In the solution of Problem 21 in Practice Examination 6, the statement under the

first expression on the right-hand side of the third to last formula should be:

number of ways to
pick ordered samples
of size 2 from
population of size n

instead of

number of ways to
pick ordered samples
of size $n-2$ from
population of size n

Posted July 17, 2010

Practice Examinations: An Introduction on page 109, the third sentence of the last section should be:

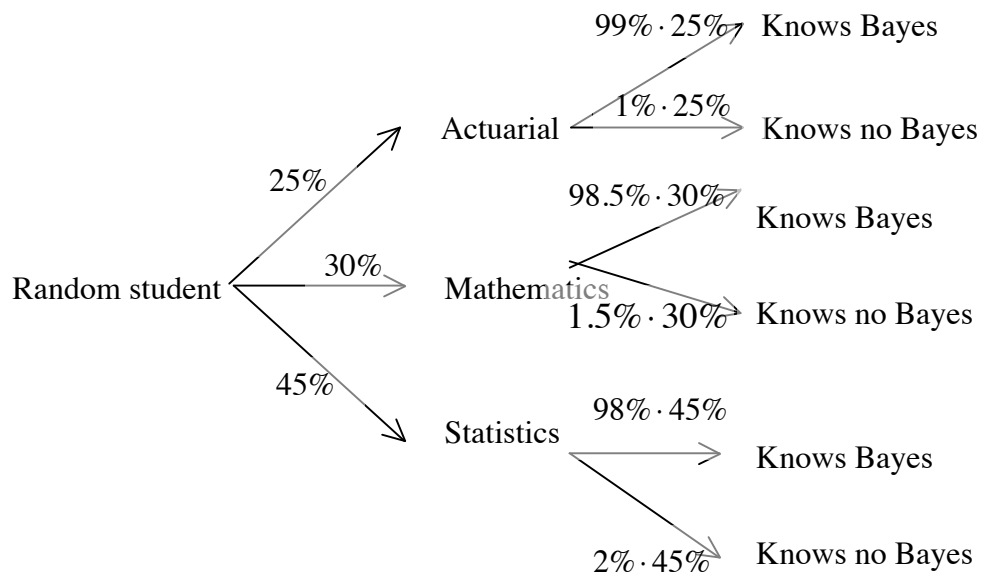
Practice examinations 6-20 are meant to be more challenging.

Posted July 3, 2010

In the solution of Problem 1, Practice Examination 5, at the end of the first part of the fourth sentence of the solution, $5/6$ is a typo, it should be $5/36$, as used in the formula for $\Pr(Y = 6)$.

Posted June 9, 2010

In the alternative solution of Problem 28, Practice Examination 11, the probability tree diagram should be:



Some numbers in the diagram were mistyped.

Posted February 25, 2010

In Problem 9, Practice Examination 20, answer D should be 0.6140, and the third to last formula should be:

$$\begin{aligned}\Pr(A^c \cap B \cap C \cap D^c) &= \Pr(A^c) \cdot \Pr(B) \cdot \Pr(C) \cdot \Pr(D^c) = \\ &= (1 - \Pr(A)) \cdot \Pr(B) \cdot \Pr(C) \cdot (1 - \Pr(D)) = 0.4 \cdot 0.5 \cdot 0.4 \cdot 0.7 = 0.056,\end{aligned}$$

while the last formula should be:

$$1 - (0.084 + 0.126 + 0.084 + 0.056 + 0.036) = 0.614.$$

Posted January 15, 2010

The last formula in the first sentence of Problem 9 in Practice Examination 20 should be $\Pr(D) = 0.3$, not $\Pr(A) = 0.3$.

Posted January 5, 2010

In the description of the gamma distribution in Section 2, the condition for the range of its MGF should be $t < \beta$, not $0 < t < \beta$.

Posted January 1, 2010

The Course P/1 syllabus updated for 2010 no longer contains direct references to chi-square, beta, Pareto, Weibull, and lognormal distributions. My interpretation of this change is that you do not need to memorize the details of chi-square, beta, Pareto and Weibull distributions, but you still should familiarize yourself with them. Since lognormal has a direct connection to normal, I think you should know that connection.

Posted November 20, 2009

Answers A and B in Problem 10, Practice Examination 9, have the symbol τ mistyped as r in the numerator, and they should be:

$$\text{A. } f_Y(y) = \frac{\tau \theta y^{\tau-1}}{(y + \theta)^{\tau+1}} \quad \text{B. } f_Y(y) = \frac{\alpha \theta^\alpha \tau y^{\tau-1}}{(y^\tau + \theta)^{\alpha+1}}$$

Posted November 17, 2009

In Problem 17 in Practice Examination 14, answer choice A should be

$$f_Y(y) = \begin{cases} 0 & y < 0, \\ e^{1-e^2} (e^{ey} + e^{-ey}) & 0 < y < e, \\ e^{1-e^2} \cdot e^{-ey} & y \geq e. \end{cases}$$

and answer choice D should be:

$$f_Y(y) = \begin{cases} 0 & y < 0, \\ e^{e^2-1} (e^{ey} + e^{-ey}) & 0 < y < e, \\ e^{e^2-1} \cdot e^{-ey} & y \geq e. \end{cases}$$

The last sentence of the solution should be:

Therefore, we can take

$$f_Y(y) = \begin{cases} 0 & y < 0, \\ e^{1-e^2} (e^{ey} + e^{-ey}) & 0 < y < e, \\ e^{1-e^2} \cdot e^{-ey} & y \geq e. \end{cases}$$

Posted September 2, 2009

In Problem 17 in Practice Examination 17, this statement

and $f_N(n+1) > f_N(n)$ for $n = 0, 1, 2, 3, 4$.

should be removed.

Posted July 9, 2009

In the solution of Problem 26 in Practice Examination 19, the formula:

$$F_{X_2}(x) = 2F_X(x) - F_{X_1}(x) = \begin{cases} 0 & x < 0, \\ 0.5x & 0 \leq x < 1, \\ 1 & x \geq 1. \end{cases}$$

should be:

$$F_{X_2}(x) = 2F_X(x) - F_{X_1}(x) = \begin{cases} 0 & x < 0, \\ 0.5x & 0 \leq x < 2, \\ 1 & x \geq 2. \end{cases}$$

Posted July 1, 2009

In Section 2, the general definition of a percentile should be

the 100- p -th *percentile* of the distribution of X is the number x_p which satisfies both of

the following inequalities: $\Pr(X \leq x_p) \geq p$ and $\Pr(X \geq x_p) \geq 1 - p$.

Posted June 19, 2009

In the discussion of the mode of Poisson distribution on page 45, the expression

and as soon $n \geq \lambda - 1$,

should be:

and as soon as $n \geq \lambda - 1$,

Posted June 18, 2009

The last formula in the solution of Problem 19 of Practice Examination 5 should be

$\Pr(X \geq 10) \leq \frac{1}{4^2} = \frac{1}{16}$ instead of $\Pr(X \geq 10) < \frac{1}{4^2} = \frac{1}{16}$.

Posted June 17, 2009

In the statement of Problem 9 in Practice Examination 7, $\Pr(X > 800)$ should be $\Pr(X \geq 800)$, and in the solution, all inequalities should be changed accordingly.

Posted May 24, 2009

Problem 7 in Practice Examination 20 needs, unfortunately, a major overhaul. My apologies. Here is what the problem should say:

The amount of damage X in a car accident is given by the exponential distribution with mean 10,000. The insurance policy covering that damage has a deductible of 500 and a policy limit of 100,000. Which of the following numbers is the closest to the coefficient of variation of the amount paid per payment (i.e., amount paid given that a payment is made by the insurance company).

- A. 1.00 B. 1.10 C. 1.20 D. 1.30 E. 1.40

Solution.

Let us write Y for the amount paid per payment. Then

$$Y = (\min(X - 500, 100000 - 500) | X > 500) = (\min(X - 500, 99500) | X > 500).$$

Therefore, for $y > 0$,

$$\begin{aligned} F_Y(y) &= \Pr(Y \leq y) = \Pr(\min(X - 500, 99500) \leq y | X > 500) = \\ &= \begin{cases} \frac{\Pr(500 < X < y + 500)}{\Pr(X > 500)}, & y < 99500, \\ 1, & y \geq 99500, \end{cases} = \\ &= \begin{cases} \frac{e^{-0.05} - e^{-0.05 - \frac{y}{10000}}}{e^{-0.05}}, & y < 99500, \\ 1, & y \geq 99500, \end{cases} = \begin{cases} 1 - e^{-\frac{y}{10000}}, & y < 99500, \\ 1, & y \geq 99500. \end{cases} \end{aligned}$$

Note that the distribution of Y is mixed, with a point mass at $y = 99500$, where CDF jumps by $e^{-9.95}$. This gives us

$$f_Y(y) = \begin{cases} \frac{1}{10000} e^{-\frac{y}{10000}}, & 0 \leq y < 99500, \\ e^{-9.95}, & y = 99500, \\ 0, & \text{otherwise.} \end{cases}$$

Therefore

$$\begin{aligned}
 E(Y) &= \int_0^{99500} \frac{ye^{-\frac{y}{10000}}}{10000} dy + 99500e^{-9.95} = \underbrace{\left[\begin{array}{ll} u = y & v = -e^{-\frac{y}{10000}} \\ du = dy & dv = \frac{1}{10000}e^{-\frac{y}{10000}} dy \end{array} \right]}_{\text{Integration by parts in the integral}} = \\
 &= -ye^{-\frac{y}{10000}} \Big|_{y=0}^{y=99500} + \int_0^{99500} e^{-\frac{y}{10000}} dy + 99500e^{-9.95} = \\
 &= -99500e^{-9.95} + 10000F_X(99500) + 99500e^{-9.95} = 10000(1 - e^{-9.95}) \approx 9999.5227.
 \end{aligned}$$

Furthermore,

$$\begin{aligned}
 E(Y^2) &= \int_0^{99500} \frac{y^2e^{-\frac{y}{10000}}}{10000} dy + 99500^2 \cdot e^{-9.95} = \underbrace{\left[\begin{array}{ll} u = y^2 & v = -e^{-\frac{y}{10000}} \\ du = 2ydy & dv = \frac{1}{10000}e^{-\frac{y}{10000}} dy \end{array} \right]}_{\text{Integration by parts in the integral}} = \\
 &= -y^2 \cdot e^{-\frac{y}{10000}} \Big|_{y=0}^{y=99500} + \int_0^{99500} 2ye^{-\frac{y}{10000}} dy + 99500^2 \cdot e^{-9.95} = \\
 &= -99500^2 \cdot e^{-9.95} + 20000 \cdot \int_0^{99500} \frac{ye^{-\frac{y}{10000}}}{10000} dy + 99500^2 \cdot e^{-9.95} = \\
 &= 20000 \cdot \int_0^{99500} \frac{ye^{-\frac{y}{10000}}}{10000} dy = 20000(E(Y) - 99500e^{-9.95}) = \\
 &= 20000 \cdot (10000(1 - e^{-9.95}) - 99500e^{-9.95}) = \\
 &= 20000 \cdot (10000 - 109500e^{-9.95}) = 10^6 \cdot (200 - 2190e^{-9.95}) \approx 199895476.
 \end{aligned}$$

Based on this

$$\text{Var}(Y) = E(Y^2) - (E(Y))^2 = 10^6 \cdot (200 - 2190e^{-9.95}) - 10^8 (1 - e^{-9.95})^2 \approx 99905021.8.$$

The coefficient of variation is

$$\frac{\sqrt{\text{Var}(Y)}}{E(Y)} = \frac{\sqrt{10^6 \cdot (200 - 2190e^{-9.95}) - 10^8 (1 - e^{-9.95})^2}}{10000(1 - e^{-9.95})} \approx 0.9996.$$

Answer A.

Posted May 17, 2009

Problem 25 in Practice Examination 18 and its solution should be:

P Sample Exam Questions, Problem No. 128, also Dr. Ostaszewski's online exercise posted November 15, 2008

An insurance agent offers his clients auto insurance, homeowners insurance and renters insurance. The purchase of homeowners insurance and the purchase of renters insurance are mutually exclusive. The profile of the agent's clients is as follows:

- i) 17% of the clients have none of these three products.
- ii) 64% of the clients have auto insurance.
- iii) Twice as many of the clients have homeowners insurance as have renters insurance.
- iv) 35% of the clients have two of these three products.
- v) 11% of the clients have homeowners insurance, but not auto insurance.

Calculate the percentage of the agent's clients that have both auto and renters insurance.

- A. 7% B. 10% C. 16% D. 25% E. 28%

Solution.

Let H be the event of a client having a homeowners insurance, R be the event of a client having a renters insurance, and A be the event of a client having an auto insurance. We are given that

$$\Pr((H \cup R \cup A)^c) = 0.17,$$

so that $\Pr(H \cup R \cup A) = 0.83$. We are also given that $H \cap R = \emptyset$, so that

$\Pr(H \cap R) = 0$. Note that $A \cap H \cap R \subset H \cap R$, so that $\Pr(A \cap H \cap R) = 0$, as well.

Also, $\Pr(A) = 0.64$, $\Pr(H) = 2\Pr(R)$, and

$$\begin{aligned} \Pr(A \cap H) - \Pr(A \cap H \cap R) + \Pr(A \cap R) - \Pr(A \cap H \cap R) + \\ + \Pr(H \cap R) - \Pr(A \cap H \cap R) = \Pr(A \cap H) + \Pr(A \cap R) = 0.35. \end{aligned}$$

Finally, we are also given that

$$\Pr(H - A) = \Pr(H - (A \cap H)) = \Pr(H) - \Pr(A \cap H) = 0.11.$$

The quantity we are looking for is $\Pr(A \cap R)$. First note that

$$0.83 = \Pr(H \cup R \cup A) = \Pr(H) + \Pr(R) + \Pr(A) -$$

This means that $\Pr(H \cap R) = 0.18$ and consequently $\Pr(H \cap A) = 0.36$. But $3\Pr(R) + 0.64 - 0.35$

$$0.36 = \Pr(H) = \Pr(H \cap A) + \Pr(H - A) = \Pr(H \cap A) + 0.11,$$

so that $\Pr(H \cap A) = 0.25$ and $\Pr(A \cap R) = 0.35 - 0.25 = 0.10$.

Answer B.

Posted April 23, 2009

In Problem 20, Practice Examination 14, the first sentence of the solution should be:

Let us define the following random variables:

X : time until death of Dwizeel by causes other than their private plane crash,

Y : time until death of Satellite Component by causes other than their private plane crash,

Z : time until death of Dwizeel and Satellite Component as a result of their private plane crash.

The solution has the words "of" mistyped as "od."

Posted April 23, 2009

In Problem 9, Practice Examination 14, the Greek letter τ in the statement of the problem should be replaced by the Greek letter α .