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Exam IFM Study Manual

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Preface

Welcome to the IFM exam!

This course combines Corporate Finance and Derivative Instruments. You will study the following topics:

1. Investment risk and project analysis
2. Efficient markets hypothesis
3. Mean-variance portfolio theory
4. Capital asset pricing model (CAPM)
5. Capital structure
6. Debt and equity financing
7. Forwards and futures
8. Call and put options
9. Strategies combining call and put options
10. Principles relating prices of calls and puts, and relating prices of options to each other and bounding them. These principles do not develop exact prices for options, but are general and easy to derive.
11. Valuing options using binomial trees.
12. The lognormal model for stocks.
13. Valuing options using analytic methods (Black-Scholes).
14. Definitions of exotic options, and pricing methods for exotic options where available.
15. Real options
16. Actuarial applications of option pricing

The IFM exam is a 3-hour exam with 30 questions. The following table gives the syllabus topics and their weights, and the lessons discussing each topic

Topic	Weight	Lessons
Mean-Variance Portfolio Theory	10–15%	5
Asset Pricing Models	5–10%	6–8
Market Efficiency and Behavioral Finance	5–10%	4,8
Investment Risk and Project Analysis	10–15%	2–3,30
Capital Structure	10%	9–13
Introductory Derivatives—Forwards and Futures	5–10%	1,14–15
General Properties of Options	10–15%	16–18
Binomial Pricing Models	10%	19–22
Black-Scholes Option Pricing Model	10–15%	23–24,27
Option Greeks and Risk Management	5–10%	25,26,31

The syllabus includes parts of two textbooks, and two study notes. Download the syllabus from the SOA site and refer to the last page of the syllabus for links to the two study notes. One textbook and one study note deal with Corporate Finance. The other textbook and study note deal with Derivative Instruments.

If you use the *Derivatives Markets* textbook, refer to the following website for errata:

http://derivatives.kellogg.northwestern.edu/errata/errata3e.html

This manual

This manual gives you complete coverage of all syllabus topics. The corporate finance topics, for the most part, come first. However, the first lesson is based on the *Derivatives Markets* textbook. This material is basic, and in particular includes the definition of shorting an asset, a concept that appears in the corporate finance part of the course, so it comes first. Also, one corporate finance topic, real options, was placed after most of the derivatives material, since, after all, a real option is an option, and one of the formulas for valuing it, Black-Scholes, is covered in the derivatives part of the course.

The two textbooks deal with interest rates differently. *Corporate Finance* uses annual effective interest rates, whereas *Derivatives Markets* uses continuously compounded interest rates. Exams will be clear as to the meaning of the interest rate they provide. But in this manual, if you're not told otherwise, assume that an interest rate discussed in a lesson in the corporate finance part is annual effective and an interest rate in the derivatives part is continuously compounded.

To help you check how much you are learning, there are quizzes within most lessons; these are straightforward exercises which you should work out as you get to them. Solutions to quizzes are at the end of the lesson, after the solutions to the exercises. The exercises at the end of each lesson are designed to be exam-like (although sometimes they are a bit long for an exam), requiring only a calculator and a normal distribution calculator (like a spreadsheet program) to solve. Working these out will help you learn the concepts.

Note the following valuable features at the end of the manual:

- Eleven practice exams.
- Solutions to relevant questions from all released exams: the Spring 2007 and Spring 2009 Exam MFE and the Spring 2007 and Fall 2007 CAS Exams 3.
- Solutions to all sample questions on derivatives. The solutions provided by the SOA are longer and often include additional commentary and educational material. My solutions get to the point and are meant to indicate the method you'd use on an exam.

  The SOA provides sample questions for Corporate Finance as well. See the syllabus for the links to the sample questions. References to these sample questions are provided at the ends of exercise sets of lessons related to those sample questions.
- A cross-reference indicating the lesson covering every relevant question from the released exams and the sample questions, and a similar cross-reference for all questions in the eleven practice exams.
- An index.

A note on notation: the McDonald textbook uses $N(x)$ to indicate the cumulative standard normal distribution at x. Many other textbooks use $\Phi(x)$ for the same concept, and in fact I use $\Phi(x)$ in the Exam LTAM, MAS-I, and STAM manuals. However, for this course, I follow McDonald and financial economics tradition and use $N(x)$.

The normal distribution table

Formulas in this course use the normal distribution. Most students will be taking this exam at a Prometric site. Prometric provides a standard normal distribution calculator. See

https://www.prometric.com/soa

to see how it works. The calculator provides values of the cumulative normal distribution function and its inverse to 5 decimal places.

When working problems in this manual, you may use Excel's `NORMSDIST` and `NORMSINV` functions to perform normal distribution calculations in a manner similar to the way you will do them on an exam. A normal distribution table is provided in Appendix D. You can use this in a pinch if you don't have access to a calculator or program that can calculate normal distribution values.

At the exam, you will have access to a formula sheet. See https://www.soa.org/Files/Edu/2018/exam-ifm-cbt-table.pdf

for this sheet. It provides the standard normal density function, the lognormal density function, moments of the lognormal distribution, and formulas for the greeks (Lesson 25).

If you are taking the exam at a paper-and-pencil site, you will be given a formula sheet and a cumulative normal distribution table. Currently, the url for this table is

https://www.soa.org/Files/Edu/2018/exam-ifm-pp-table.pdf

The formula sheet has rules for the use of the table, indicating that you should not interpolate in the table; use rounded values. Read those rules for more details.

Since most students will be taking the exam under CBT, this manual uses the more exact method of calculating normal distribution values, namely 5-place precision.

Helpful links

Download the syllabus for the exam, which is found at

https://www.soa.org/education/exam-req/edu-exam-ifm-detail/

In this manual, questions related to each lesson are listed at the lesson. The document containing the Finance and Investment questions is at

https://www.soa.org/globalassets/assets/Files/Edu/2018/2018-exam-ifm-sample-questions.pdf

The document containing the Introductory and Advanced Derivatives questions is at

https://www.soa.org/globalassets/assets/Files/Edu/2018/ifm-derivatives-questions-solutions.pdf

However, the Introductory Derivatives questions are already incorporated in the manual's exercises.

Future of this exam

The Society of Actuaries has announced that this exam will not be part of the syllabus after 2022. The last administration of this exam will be in November 2022.

Students who pass this exam will be exempt from the ATPA exam in the new syllabus. Note that there is no relationship between the topics on this exam and the topics on ATPA.

The SOA also notes that a small number of topics from this exam will be moved to the new ALTAM exam that will start to be administered in 2023. So if you unfortunately do not pass IFM by the end of 2022, some of what you learned may still be useful for the new syllabus. My wild guess is that the new ALTAM will include very basic knowledge of put-call parity, binomial trees, and Black-Scholes.

The CAS hasn't made its final intentions known, but my guess is that the CAS will follow the SOA and will drop 3F from its syllabus.

Errata

Please report all errors to the author. You may send them to the publisher at mail@studymanuals.com or directly to me at errata@aceyourexams.net.

An errata list will be posted at http://errata.aceyourexams.net. Check this errata list frequently.

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Lesson 2

Project Analysis

Reading: *Corporate Finance* 8.5, IFM 21-18 2–3

This lesson begins the corporate finance part of the course. From here up to and including Lesson 13, when not told otherwise, assume that interest rates are annual effective.

2.1 NPV

When a company considers embarking on a project, it must verify that this project will meet the company's financial goals. The measure we will use for this is ***NPV***, or net present value. To compute the NPV we calculate the *free cash flows* of the project. The free cash flows are the cash amounts generated by the project itself, both positive and negative, year by year. Cash flows do not include non-cash accounting items, such as depreciation.[1] The free cash flows also do not include cash flows from financing used to support the project. If a loan is taken to pay the project's initial expenses, neither the loan nor interest on the loan is part of free cash flows. Free cash flows are purely cash generated by the project itself.

The NPV is the present value, at the start of the project, of the project's free cash flows. At what interest rate is the NPV calculated? Usually the NPV is calculated at the interest rate the company must pay to finance the project. In other words, the NPV is calculated at the interest rate that has to be paid to investors in order to get them to invest in the project. This interest rate is called the ***cost of capital***. We will discuss what the cost of capital should be in the following lessons.

EXAMPLE 2A A life insurance company is considering developing a new Universal Life product. It will cost \$5 million, payable immediately, to develop this product, and developing the product will take a year. The company estimates free cash flows will be \$−1 million at the end of the first year, followed by \$1 million per year for 5 years, and will then decrease at a compounded rate of 10% per year after that.

The company's cost of capital is 12%.

Calculate the NPV of the project. ■

SOLUTION: The following time line shows the cash flows in millions:

\$−5	\$−1	\$1	\$1	\$1	\$1	\$1	\$0.9	\$0.81	• • •
0	1	2	3	4	5	6	7	8	

The NPV generated during the first 6 years is

$$-5{,}000{,}000 - \frac{1{,}000{,}000}{1.12} + 1{,}000{,}000\left(\frac{a_{\overline{5}|0.12}}{1.12}\right) = -5{,}892{,}857 + 1{,}000{,}000\left(\frac{1-1/1.12^5}{0.12(1.12)}\right) = -2{,}674{,}307$$

After 6 years, free cash flows form a geometric series with first term $900{,}000/1.12^7$ and ratio $0.9/1.12$. The NPV generated after year 6, in millions, is

$$900{,}000\left(\frac{1/1.12^7}{1-0.9/1.12}\right) = 2{,}072{,}582$$

Total NPV is $-2{,}674{,}307 + 2{,}027{,}582 = \boxed{\mathbf{-601{,}725}}$. □

[1]However, for insurance products, they include changes in reserves. A company must set aside cash to support the reserves, although this cash may be invested.

Companies should invest in a project only if NPV > 0. Otherwise they destroy the value of the company.

If we assume that free cash flows are constant, they form a **perpetuity**. As you learned in Financial Mathematics, the present value of an immediate perpetuity of 1 per year is $1/i$, where i is the interest rate. If the free cash flows are 1 in the first year and grow at compounded rate g, then their present value is

$$\begin{aligned}\mathrm{NPV} &= \sum_{n=1}^{\infty}\frac{(1+g)^{n-1}}{(1+i)^n}\\ &= \frac{1/(1+i)}{1-(1+g)/(1+i)}\\ &= \frac{1/(1+i)}{(i-g)/(1+i)} = \frac{1}{i-g}\end{aligned} \tag{2.1}$$

Quiz 2-1 A company is considering a project. This project will require an investment of 10 million immediately and will generate free cash flows of 1 million per year at the end of one year, increasing at a compounded rate of 3% per year perpetually.

The cost of capital is 9%.

Calculate the NPV of the project.

2.2 Project analysis

2.2.1 Break-even analysis

Companies analyze the risk in a project. One way to analyze the risk is to vary the assumptions used to calculate the NPV with the changed assumptions. ***Break-even analysis*** consists of determining the value of each assumption parameter for which the NPV is 0, assuming that the other assumption parameters are at their baseline values.

Calculation of **IRR** is an example of break-even analysis. IRR, the internal rate of return, is an alternative profit measure to NPV. The IRR is the interest rate r such that the present value of free cash flows at r is 0. Assuming the usual pattern of negative free cash flows initially followed by positive free cash flows, IRR is the highest interest rate for which the NPV is at least 0. Thus IRR is the highest interest rate for which the company breaks even.

A similar analysis can be done for the other parameters. A break-even analysis calculates the break-even level of number of sales, expenses, sales price, level of cash flows per year, and any other parameter.

EXAMPLE 2B A project requires an immediate investment of 19 million. It is expected to generate free cash flows of 2 million per year at the end of the first year, growing 2% per year perpetually. The cost of capital is 12%.

Perform a break-even analysis on the rate of growth of free cash flows. ■

SOLUTION: Let g be the growth rate. We want to solve $-19 + \frac{2}{0.12-g} = 0$

$$\begin{aligned}-19 + \frac{2}{0.12-g} &= 0\\ \frac{2}{0.12-g} &= 19\\ 0.12-g &= \frac{2}{19}\\ g &= \boxed{\mathbf{0.01474}}\end{aligned}$$ □

Quiz 2-2 A project to develop a new product requires an immediate investment of 9 million. It will then generate free cash flows of 1 million per year starting with the end of the first year, until the product becomes obsolete and cannot be sold. The cost of capital is 10%.

Perform a break-even analysis on the number of years the product must sell.

2.2.2 Sensitivity analysis

Sensitivity analysis consists of calculating the change in the NPV resulting from a change in a parameter. Typically one sets the parameter to its value in the worst possible case and the best possible case, and calculates the NPV for both cases. This analysis shows which parameters have the greatest impact on the NPV.

Example 2C A project to develop a new product requires an immediate investment of 15 million. Free cash flows generated by this project are 20% of sales. Sales are expected to be level, and to continue for a certain number of years, at which point the product becomes obsolete. The best and worst cases for each assumption are:

	Worst case	Baseline	Best case
Annual sales ($ million)	20	25	30
Number of years	3	5	7
Cost of capital	0.16	0.12	0.08

Perform a sensitivity analysis on the three factors listed in the table. Which factor is the NPV most sensitive to? ■

Solution: We'll do all calculations in millions.

For annual sales s:

$$\text{NPV} = -15 + 0.2s\,a_{\overline{5}|0.12} = -15 + 0.2s\left(\frac{1 - 1/1.12^5}{0.12}\right)$$

which is −\$0.581 million for $s = 20$ and \$6.629 million for $s = 30$, a variation of \$7.210 million

For number of years n:

$$\text{NPV} = -15 + 5a_{\overline{n}|0.12} = -15 + 5\left(\frac{1 - 1/1.12^n}{0.12}\right)$$

which is −\$2.991 million for $n = 3$ and \$7.818 million for $n = 7$, a variation of \$10.809 million.

For cost of capital r:

$$\text{NPV} = -15 + 5a_{\overline{5}|r} = -15 + 5\left(\frac{1 - 1/(1 + r)^5}{r}\right)$$

which is \$1.371 million for $r = 0.16$ and \$4.964 for $r = 0.08$, a variation of \$3.593 million.

We see that number of years of sales is the assumption to which NPV is most sensitive. □

2.2.3 Scenario analysis

Often parameters are correlated and should not be analyzed separately. For example, increasing the price of a product may lower sales. **Scenario analysis** consists of calculating the NPV for various scenarios. A scenario may vary two parameters in a consistent manner, leaving the other parameters unchanged if they are uncorrelated.

2.3 Risk measures

In the previous section we analyzed risk by varying parameters. An alternative method for analyzing risk is to assign a number to the project indicating its riskiness. This section discusses such risk measures. Each of these **risk measures** is a function from a random variable to a real number. The random variable may be profits, returns on investment, or aggregate loss amounts paid by an insurance company. Notice that the direction of risk for aggregate loss amounts is the opposite of profits or returns: the risk is that profits or returns are low and that loss amounts are high.

2.3.1 Four risk measures

We will discuss four risk measures: variance, semi-variance, VaR, and TVaR.

Variance

Variance is a popular risk measure and will be used in **mean-variance portfolio theory**, which we discuss starting in Lesson 5. If R is the random variable for the return on an investment, the mean return is μ and the variance is

$$\operatorname{Var}(R) = \sigma^2 = \mathbf{E}[(R-\mu)^2] = \mathbf{E}[R^2] - \mu^2 \tag{2.2}$$

An equivalent risk measure is the square root of the variance, or the **standard deviation** σ. We'll also use the notation $\operatorname{SD}(R)$ for the standard deviation.

$$\operatorname{SD}(R) = \sqrt{\operatorname{Var}(R)} = \sigma$$

The standard deviation of the rate of return is also called the *volatility* of the rate of return.

The variance may be estimated from a sample using the formula

$$\hat{\sigma}^2 = \sum_{i=1}^{n} \frac{(R_i - \bar{R})^2}{n}$$

where $\bar{R}$ is the **sample mean**. That is the formula given in the study note, but usually the denominator is $n-1$ instead of n to make this estimate unbiased. In fact, the formula for estimating volatility given in the Berk/DeMarzo textbook is equation (5.1) on page 59, and that formula divides by $n-1$.

Semi-variance

Since we are more concerned with underperformance than overperformance, at least for profits and rates of return on investments, we may prefer the downside semi-variance, which we'll refer to as the ***semi-variance*** for short, as a measure of risk. The semi-variance considers the square difference from the mean only when that difference is negative. It is defined by

$$\sigma_{SV}^2 = \mathbf{E}\Big[\min\big(0, (R-\mu)\big)^2\Big] \tag{2.3}$$

The semi-variance is positive even though it is based on negative differences from the mean, since the differences are squared. The square root of the downside semi-variance is the downside standard deviation.

Example 2D The random variable X has an exponential distribution with mean 1:

$$f_X(x) = e^{-x}, \qquad x > 0$$

Calculate the semi-variance of X. ■

Solution: We integrate $\min(0, x-1)^2$ over the density function. This minimum is 0 for $x > 1$, so we only integrate up to 1. We'll integrate by parts twice.

$$\begin{aligned}
\int_0^1 (x-1)^2 e^{-x}\,dx &= -(x-1)^2 e^{-x}\Big|_0^1 + 2\int_0^1 (x-1)e^{-x}\,dx \\
&= 1 + 2\left(-(x-1)e^{-x}\Big|_0^1 + \int_0^1 e^{-x}\,dx\right) \\
&= 1 - 2 + 2(1-e^{-1}) = 1 - 2e^{-1} = \boxed{\mathbf{0.264241}}
\end{aligned}$$

□

The sum of the downside semi-variance and the upside semi-variance is the variance:

$$\begin{aligned}\mathbf{E}[(X-\mu)^2] &= \int_{-\infty}^{\infty}(x-\mu)^2 f_X(x)\,\mathrm{d}x\\ &= \int_{-\infty}^{\mu}(x-\mu)^2 f_X(x)\,\mathrm{d}x + \int_{\mu}^{\infty}(x-\mu)^2 f_X(x)\,\mathrm{d}x\end{aligned}$$

The first term of the last expression is the downside semi-variance and the second term is the upside semi-variance.

The semi-variance may be estimated from a sample by

$$\hat{\sigma}_{SV}^2 = \frac{1}{n}\sum_{i=1}^{n}\min\bigl(0,(R_i-\bar{R})\bigr)^2 \tag{2.4}$$

Quiz 2-3 You are given the following sample:

5 10 15 20 25

Calculate the sample semi-variance.

Value-at-Risk (VaR)

The **VaR** of a random variable X at level α is the 100α percentile of the random variable. For a continuous random variable, it is x such that $\Pr(X \le x) = \alpha$. For profits or rates of return, where the risk is that X is low, α is picked low, with values like 0.05, 0.025, 0.01, 0.005. For aggregate insurance losses, where the risk is that X is high, α is picked high, with values like 0.95, 0.975, 0.99, 0.995.

The VaR is calculated by inverting the cumulative distribution function:

$$\mathrm{VaR}_\alpha(X) = F_X^{-1}(\alpha) \tag{2.5}$$

Example 2E Profits have a distribution with the following density function:

$$f(x) = \frac{3}{(1+x)^4} \qquad x > 0$$

Calculate VaR of profits at the 0.01 level. ■

Solution: Integrate $f(x)$ to obtain the cumulative distribution function, then invert that function at 0.01.

$$\begin{aligned}F(x) &= \int_0^x \frac{3\,\mathrm{d}t}{(1+t)^4} = 1 - \frac{1}{(1+x)^3}\\ 1-\frac{1}{(1+x)^3} &= 0.01\\ 1+x &= \sqrt[3]{1/0.99} = 1.003356\\ x &= \boxed{\mathbf{0.003356}}\end{aligned}$$

□

To estimate VaR from a sample, the sample is ordered from lowest to highest, and then the 100α percentile is selected. This percentile is not well-defined since a sample is a discrete distribution, so some rule for selecting the percentile is needed. For example, if the sample is size 1000 and $\alpha = 0.05$, then one might set the sample VaR equal to the 50^{th} order statistic (most conservative), the 51^{st} order statistic, or some weighted average of the two, such as the smoothed empirical percentile defined in the Exam STAM syllabus.

Tail value-at-risk

While value-at-risk identifies the amount which returns or profits will exceed a great proportion ($1-\alpha$ to be exact) of the time, it doesn't consider the severity of the downside risk in the remaining α of the time. **Tail value-at-risk**, also known as Conditional Tail Expectation (CTE) or Expected Shortfall measures this risk. It is defined as the expected value of the random variable given that it is below the 100α percentile for downside risk

$$\mathrm{TVaR}_\alpha(X) = \mathbf{E}[X \mid X < \mathrm{VaR}_\alpha(X)] = \frac{\int_{-\infty}^{\mathrm{VaR}_\alpha(X)} x f(x)\,\mathrm{d}x}{\alpha} \tag{2.6}$$

or the expected value of the random variable given that it is above the 100α percentile for upside risk, like aggregate losses

$$\mathrm{TVaR}_\alpha(X) = \mathbf{E}[X \mid X > \mathrm{VaR}_\alpha(X)] = \frac{\int_{\mathrm{VaR}_\alpha(X)}^{\infty} x f(x)\,\mathrm{d}x}{1-\alpha} \tag{2.7}$$

You should be able to figure out whether upside or downside risk is present based on what is being analyzed, but if not, if $\alpha < 0.5$, presumably the risk is downside and if $\alpha > 0.5$, presumably the risk is upside.

It may be difficult or impossible to evaluate the intergral needed to calculate TVaR.

TVaR can be estimated from a sample. Select the bottom or top α proportion of the items of the sample and calculate their mean. For example, if the sample is size 1000 and $\alpha = 0.05$, average the lowest 50 items of the sample to calculate downside risk.

2.3.2 Coherent risk measures

Let's list four desirable properties of a risk measure $g(X)$.

1. **Translation invariance.** Adding a constant to the random variable should add the same constant to the risk measure. Or:
$$g(X+c) = g(X) + c$$
This is reasonable, since a constant gain or loss generates no risk beyond its amount.

2. **Positive homogeneity.** Multiplying the random variable by a positive constant should multiply the risk measure by the same constant:
$$g(cX) = cg(X)$$
This is reasonable, since expressing the random variable in a different currency (for example) should not affect the risk measure.

3. **Subadditivity.** For any two random losses X and Y, the risk measure for $X+Y$ should not be greater than the sum of the risk measures for X and Y separately:
$$g(X+Y) \le g(X) + g(Y)$$
This is reasonable, since combining losses may result in diversification and reducing the total risk measure, but it should not be possible by breaking a risk into two sub-risks to reduce the total risk measure.

   This is for measuring upside risk. For measuring downside risk, the subadditivity property becomes $g(X+Y) \ge g(X) + g(Y)$.[2]

4. **Monotonicity.** For any two random losses X and Y, if X is always less than Y, or even if the probability that X is less than or equal to Y is 1, then the risk measure for X should be no greater than the risk measure for Y.
$$g(X) \le g(Y) \text{ if } \Pr(X \le Y) = 1$$
This is reasonable, since X clearly has no more risk than Y.

   This is for measuring upside risk. For measuring downside risk, the monotonicity property becomes $g(X) \ge g(Y)$ if $\Pr(X \ge Y) = 1$.[2]

Risk measures satisfying all four of these properties are called ***coherent***.

Risk measures with variance in their formula (such as variance itself and **semi-variance**) fail the monotonicity

[2]The study note does not mention that the inequalities for coherence for downside risks are reversed.

Table 2.1: Formula Summary

NPV

$$\mathrm{NPV} = \sum_{n=0}^{\infty} \frac{\mathrm{FCF}_n}{(1+r)^n}$$

where

FCF_n is free cash flow at time n
r is the cost of capital

If free cash flows are k at time 1 and grow at constant rate g, and the cost of capital is r, then their NPV is

$$\frac{k}{r-g} \tag{2.1}$$

Downside semi-variance:

$$\sigma_{SV}^2 = \mathbf{E}\left[\min\left(0, (R-\mu)\right)^2\right] \tag{2.3}$$

Sample downside semi-variance:

$$\hat{\sigma}_{SV}^2 = \frac{1}{n}\sum_{i=1}^{n} \min\left(0, (R_i - \bar{R})\right)^2 \tag{2.4}$$

Value-at-risk:

$$\mathrm{VaR}_\alpha(X) = F_X^{-1}(\alpha) \tag{2.5}$$

TVaR for downside risk:

$$\mathrm{TVaR}_\alpha(X) = \mathbf{E}[X \mid X < \mathrm{VaR}_\alpha(X)] = \frac{\int_{-\infty}^{\mathrm{VaR}_\alpha(X)} x f(x)\,\mathrm{d}x}{\alpha} \tag{2.6}$$

TVaR for upside risk:

$$\mathrm{TVaR}_\alpha(X) = \mathbf{E}[X \mid X > \mathrm{VaR}_\alpha(X)] = \frac{\int_{\mathrm{VaR}_\alpha(X)}^{\infty} x f(x)\,\mathrm{d}x}{1-\alpha} \tag{2.7}$$

property, since a constant has less variance than a random variable that varies, even if the random variable is always less than the constant.

Value-at-risk is not subadditive and therefore not coherent, but **tail value-at-risk** is coherent. Value-at-risk satisfies the other properties. In special cases, such as when all distributions under consideration are normal, value-at-risk is coherent.

Exercises

2.1. A project requires an immediate investment of 12 million and an additional investment of 1 million per year for 5 years starting at the end of year 1. The project will generate free cash flows (ignoring the investment cash flows) of 1.5 million in year 1, growing 2% per year perpetually. The cost of capital is 10%.

Calculate the NPV of this project.

Exercises continue on the next page . . .

2.2. A project to produce new widgets requires a \$10 million investment paid immediately. Installing the machinery will take one year, during which time no widgets will be sold. It is expected that the sale of widgets will generate \$2 million of free cash flows in year 2, growing \$200,000 per year until year 11, at which time they will become obsolete and will not be sold any more.

The cost of capital is 10%.

Calculate the NPV of this project.

2.3. A project to produce desks requires an investment of \$20 million immediately. The machinery will last for 7 years, at which point the project ends. You are given:

(i) The desks will sell for \$500 apiece.
(ii) The same number of desks will be sold each year.
(iii) There will be fixed costs of \$1 million per year, and the variable costs associated with manufacturing and selling the desks are \$200 apiece.
(iv) The revenues from selling the desks and the associated fixed and variable costs occur at the end of each year.
(v) The cost of capital is 12%.

Based on a break-even analysis, calculate the number of desks per year that must be sold.

2.4. A project requires an immediate investment of 8 million. An additional investment of 2 million is required at the end of year 1. Starting in the second year, the project will generate free cash flows of 1 million per year, growing 3% per year perpetually.

Based on a break-even analysis, determine the cost of capital to break even.

2.5. A project requires an investment of 8 million. The following are baseline, best case, and worst case assumptions:

	Worst case	Baseline	Best case
Free cash flows in first year	1.1	1.2	1.3
Rate of growth of free cash flows	0	0.03	0.05
Number of years of free cash flows	7	10	13

The cost of capital is 0.10.

Which of the three assumptions in the table is the NPV most sensitive to?

2.6. A company invests 8 million in a project to produce a new product. The product can be perpetually. A sensitivity analysis considers the following assumptions:

	Worst case	Baseline	Best case
Annual number of units sold	1,000,000	1,200,000	1,500,000
Price per unit	1.25	1.50	1.60
Expenses, as percentage of sales price	23%	20%	15%

The cost of capital is 0.15.

To which assumption is the NPV most sensitive?

Exercises continue on the next page . . .

2.7. You are given the following sample:

$$1 \qquad 3 \qquad 7 \qquad 15 \qquad 25 \qquad 39$$

Calculate the downside semi-variance.

2.8. A random variable X follows a normal distribution with $\mu = 20$, $\sigma^2 = 100$.
Calculate the downside standard deviation of X.

2.9. A random variable X has the following probability density function:

$$f(x) = \begin{cases} 2x & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

Calculate the downside semi-variance of X.

2.10. Profits X have the following cumulative distribution function:

$$F(x) = e^{-1000/x} \qquad x > 0$$

Calculate the value-at-risk at 1%.

2.11. Profits X have the following cumulative distribution function:

$$F(x) = \begin{cases} 1 - \left(\dfrac{1000}{x}\right)^2 & x > 1000 \\ 0 & \text{otherwise} \end{cases}$$

Calculate the value-at-risk at 0.5%.

2.12. Losses on an insurance are distributed as follows:

Greater than	Less than or equal to	Probability
0	1000	0.45
1000	2000	0.25
2000	5000	0.22
5000	10000	0.05
10000	20000	0.03

Within each range losses are uniformly distributed.
Calculate the tail value-at-risk for losses at 95%.

2.13. Profits X have the following cumulative distribution function:

$$F(x) = 1 - e^{-x/1000} \qquad x > 0$$

Calculate the tail value-at-risk at 5%.

2.14. For a simulation with 100 runs, the largest 20 values are

920	920	922	925	926	932	939	940	943	945
948	952	959	962	969	976	989	1005	1032	1050

Estimate TVaR at 95% from this sample.

Exercises continue on the next page . . .

2.15. Consider the risk measure $g(X) = \mathbf{E}[X^2]$. Assume it is used only for nonnegative random variables. Which coherence properties does it satisfy?

2.16. Consider the risk measure $g(X) = \mathbf{E}[\sqrt{X}]$. Assume it is used only for nonnegative random variables. Which coherence properties does it satisfy?

Finance and Investment sample questions[3]: 27,34,35,42

Solutions

2.1. The present value of the investment is $12 + (1 - 1/1.1^5)/0.1 = 15.791$ million. The present value of the free cash flows is $1.5/(0.1 - 0.02) = 18.75$ million. The NPV is $18.75 - 15.791 = \boxed{\textbf{2.959 million}}$.

2.2. At time 1, the present value of the cash flows from the widgets is $1{,}800{,}000a_{\overline{10}|} + 200{,}000(Ia)_{\overline{10}|}$.

$$\begin{aligned}
a_{\overline{10}|} &= \frac{1 - 1/1.1^{10}}{0.1} = 6.144567\\
\ddot{a}_{\overline{10}|} &= (6.144567)(1.1) = 6.759024\\
(Ia)_{\overline{10}|} &= \frac{6.759024 - 10/1.1^{10}}{0.1} = 29.03591
\end{aligned}$$

So the present value of the cash flows at time 1 is $6.144567(1{,}800{,}000) + 29.03591(200{,}000) = 16{,}867{,}404$. Discounting to time 0 and subtracting the investment, the NPV is $16{,}867{,}403/1.1 - 10{,}000{,}000 = \boxed{\textbf{\$5,334,002}}$.

2.3. Present value of investment and fixed expenses is

$$20 + a_{\overline{7}|} = 20 + \frac{1 - 1/1.12^7}{0.12} = 24.563757 \text{ million}$$

Present value of net profit from sale of 1 desk per year is

$$300\left(\frac{1 - 1/1.12^7}{0.12}\right) = 1369.127$$

So to break even, $24{,}563{,}757/1369.127 = \boxed{\textbf{17,941}}$ desks per year must be sold.

2.4. Let r be the cost of capital. At time 1, the present value of future free cash flows is $1/(r - 0.03)$ in millions. Thus at time 0 the present value of these cash flows is $1/\big((1+r)(r - 0.03)\big)$. We want r such that

$$\begin{aligned}
-8 - \frac{2}{1+r} + \frac{1}{(1+r)(r-0.03)} &= 0\\
-8(1+r)(r-0.03) - 2(r-0.03) + 1 &= 0\\
8r^2 + 9.76r - 1.3 &= 0\\
r &= \boxed{\mathbf{0.121163}}
\end{aligned}$$

[3]See the preface for the link to the document containing these questions.

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