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Exam MAS-I Study Manual

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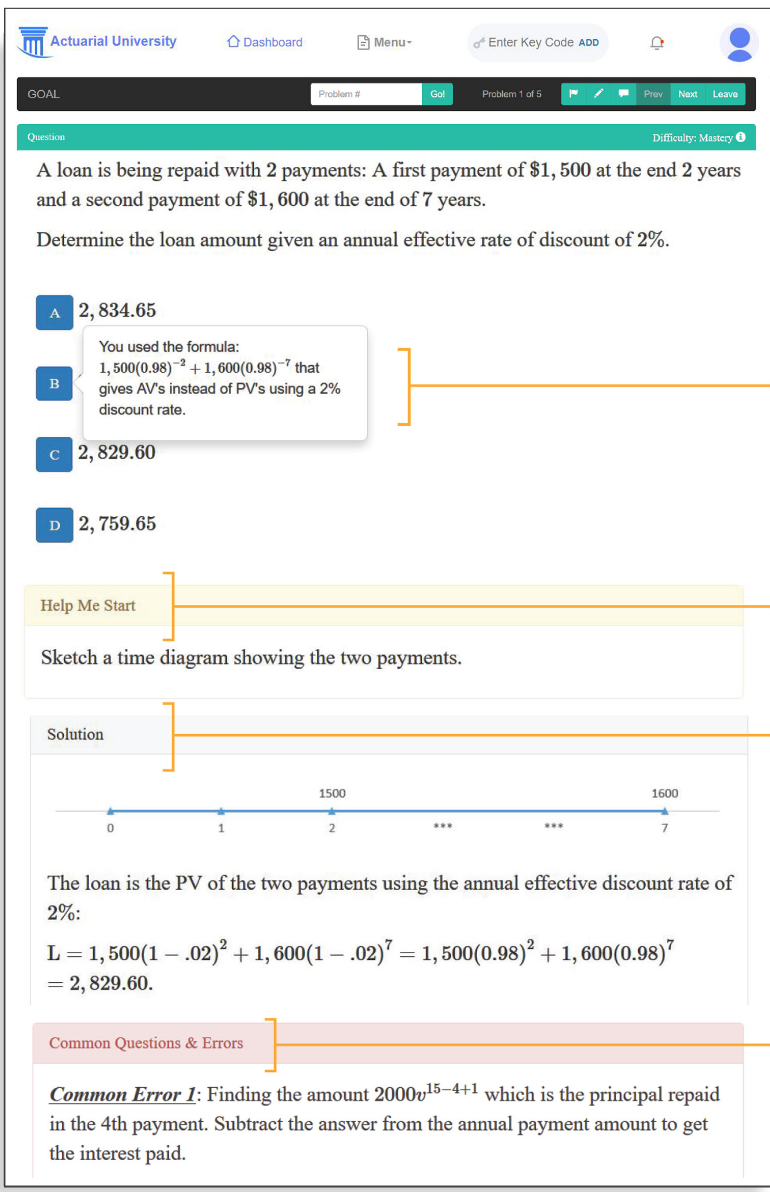

Flag for review, record notes & email the professor

Monitor difficulty level

Got it wrong? Often there is a simple reason why

Helpful strategies to get you started

Graphs and other solution techniques demonstrated when applicable

Commonly encountered errors

Start Your Exam Prep Today!

Introduction

Welcome to Exam MAS-I!

Syllabus

Exam MAS- I is a 4 hour exam. Every released Exam MAS-I had 45 questions, so I expect it to continue to have 45 questions.

At this writing, the syllabus is posted at https://www.casact.org/sites/default/files/2021-11/ExamMASI_Syllabus.pdf.

The exam has the following major topics:

- Probability models: Poisson processes, Markov chains, reliability theory, and life contingencies, simulation
- Statistics
- Extended linear models
- Time series with constant variance

The following table gives the weights on the topics from the syllabus, and number of questions on the two released exams.

	MAS-I Syllabus Weight	Spring 2018	Fall 2018	Spring 2019	Fall 2019
Probability	0%	0	1	2	1
Poisson processes	2–18%	6	4	5	4
Reliability theory	2–8%	4	3	1	3
Markov chains	2–8%	2	3	3	2
Life contingencies	2–5%	2	3	2	2
Simulation	2–5%	1	1	2	1
Est quality, kernel	5–10%	1	3	3	3
Parameter estimation		2	3	3	2
Hypothesis testing	5–20%	5	4	5	5
Generalized linear model	30–50%	11	8	8	7
Statistical learning		6	7	6	9
Time series	10–20%	5	5	5	5
Total		45	45	45	45

The CAS syllabus is difficult to interpret. For example:

- Learning objective A4 says "Apply Poisson process concepts to calculate the hazard function and related survival model concepts. . . " and gives Ross 5.2 as the reading. However, Ross 5.2 does not introduce Poisson processes (they get introduced Section 5.3) and does not discuss survival models. I've included this learning objective as "Poisson processes" in the above table (it is worth 2 questions), but I wonder whether that is appropriate.
- Learning objective A5 refers to various life contingency concepts, such as joint life, last survivor, multiple decrement, and gives Ross 9.1–9.6 as the reading. Ross does not use any of this terminology. Chapter 9 of Ross discusses reliability theory. Apparently what the CAS had in mind is that the Ross material could be used to evaluate life contingency questions, even though Ross doesn't discuss that particular application.

Tables

Download the tables you will get at the exam from the CAS website. You will need them to work out exercises in this manual. These tables list characteristics such as density functions and moments for many common distributions. The tables also include the CDF of the standard normal distribution, critical values for the t, F, and chi-square distributions.

The tables are found at

https://live-casact.pantheonsite.io/sites/default/files/2021-03/masi_tables.pdf.

On the released exams, the following instructions were given regarding using the normal distribution table:

When using the normal distribution, unless told otherwise, do not interpolate. If you want $\Phi(x)$, round x to two places and use the value in the table. For example, $\Phi(1.1342)$ would be evaluated as 0.8708. If you want $\Phi^{-1}(z)$, find the 4-digit value in the table that is closest to z and use its inverse. For example, $\Phi^{-1}(0.92) = 1.41$. The exception to this rule is that the 3-digit values for special values of z shown at the bottom of the table should be used when appropriate. For example, for the 90^{th} percentile of a standard normal distribution use 1.282, not 1.28.

Currently, you get a spreadsheet with Excel functions at the exam. Using `NORMSDIST` and `NORMSINV` you can obtain normal values to high degrees of precision. So I am not sure the previous paragraph is still applicable. In this manual, the method of the previous paragraph will be used. However, on the exam, read the exam instructions carefully and follow them.

Characteristics of CAS exams

This may be the first CAS exam you are taking. CAS exams have a somewhat different style from SOA or the former jointly sponsored exams.

CAS exams have a guessing penalty, so omit questions unless you can at least eliminate some choices.

CAS exams usually provide ranges rather then specific answer choices. The ranges are usually equal in size, and your answer should usually not be more than the size of a range lower than the first choice or higher than the last choice. For example, if the choices offered are

A. Less than 5
B. At least 5, but less than 7
C. At least 7, but less than 9
D. At least 9, but less than 11
E. At least 11

then your answer should not be less than 3 or more than 13. However, this rule is not hard-and-fast, so if you get an answer far out of range, it is suspect but not necessarily wrong.

Before 2020, every CAS exam was released, and answer choices were provided as well, but not worked out solutions. Exams are now administered with computer-based testing, so starting in 2020, exams will not be released.

Released CAS exams frequently had defective questions. If you look at the old Exam 3, Exam 3L, Exam ST, Exam LC, and Exam S answer lists, you will frequently find that more than one answer was accepted. There was an average of one defective question per exam. These defects were of the following types:

1. Questions with typos. These questions are usually not considered defective; the typo is annoying, but you still have to work out the question.

2. Questions with poor or incorrect wording. In these questions, you have to figure out what they meant and answer the question accordingly, rather than interpret the question literally.

3. Questions with ambiguous wording allowing more than one interpretation. These are the questions where often more than one answer choice is accepted.

4. Questions with specific answer choices (rare for the CAS) in which none of the answer choices is correct. Or more commonly, answers with range answer choices in which the answer is far out of the range. For example, a question with ranges like < 10, 10–30, 30–50, 50–70, and > 70 where the answer is 822.

5. Questions that cannot be answered with the information and tables that you are given.

With the move to computer-based testing and reused questions, perhaps questions will no longer be defective. But if you have great difficulty solving a question despite knowing the underlying material, it may be best to move on to the next question.

The exam spreadsheet

At the exam you will be given a spreadsheet. For a sample, go to

https://home.pearsonvue.com/cas

and at the bottom of the webpage, click "SAMPLE SPREADSHEET". This spreadsheet includes a TI-30XS calculator.

The CAS has stated that the computational nature of exam questions will not change for the meantime. Questions will not require use of spreadsheet functions to solve.s

Features of this manual

This manual has about 1300 exercises; about 600 of them are original. The exercises taken from old exams are indicated by xxx–yy:zz where xxx is the exam number or name, yy is the date of the exam (e.g. S14=spring 2014, F18=fall 2018), and zz is the question number. Almost no exercises were taken from recent exams, but a list of relevant questions from recent exams is given at the end of each lesson, and solutions to these questions can be found in Appendix B.

In lessons that deal with topics that have been on the syllabus for a long time, there are lots of exercises, mostly old exam questions. In lessons dealing with topics new to the syllabus, or that were on the syllabus in the past only briefly, most or all exercises are original and there are only a few of them. Moreover, there can be no guarantee that exam questions will emphasize the same topics as the exercises. Whenever a lesson has only a few exercises, do all of them.

Solutions to all relevant questions from old Exams 3L, LC, ST, S, and MAS-I are in Appendix B.

This manual has an index.

This manual, whose primary purpose is exam preparation, does not cover `R`. Exams do not expect you to program in `R`; they merely expect you to understand outputs from programs. I believe the material in this manual will be sufficient for you to work out any exam qu estion. On the other hand, applying the methods discussed in this course to real life requires running computer programs. If you are using this manual as a textbook replacement, I encourage you to go through the labs in *An Introduction to Statistical Learning* in order to learn how to use `R` to carry out the methods discussed in this course.

New to this edition

The CAS redesigned their website in early 2021, and removed all pre-2011 exams, as well as the Spring 2016 LC and ST exams. This edition therefore incorporates relevant old questions from those exams in the exercises.

At the beginning of Appendix C, there is a discussion on the relevance of the old exams to the current syllabus. The pre-2011 exams provide relevant questions for Poisson processes and statistics, and questions of limited usefulness for Lesson 4 and basic normal models. They also have many questions on life contingencies, but life contingencies is a very small part of the current syllabus so those questions were not added to this edition.

Acknowledgements

I would like to thank the SOA and CAS for allowing me to use their old exam questions.

The creators of TEX, LATEX, and its multitude of packages all deserve thanks for making possible the professional typesetting of this mathematical material.

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I'd like to thank the following readers who submitted errata for this manual: Adam Karnik, Megan Benoit, Jonathan Brand, Joseph Breslin, Wenqi Chen, Deb Clough, Jorge Miguel Conceicao, Zack de la Pena, Reginald Dorsey, 4Janeth Fernández, Christopher Filips, Nehama Florans, Weston Hogan, Aaron Johnson, Adam Karnik, Tzong Her Lee, Matthew Lenko, Dianting Liu, Lawrence Miley, Jaime Mullen, Li Kee Ong, Dauna Papakirykos, Seth Reuter, Alyssa Rineha4rt, Juan Sancen-Bravo, Adam Tolnay, Aytan Wachspress, Woosuk Yoo.

Errata

Please report all errors you find in this manual to the author. You may send them to the publisher at `mail@studymanuals.com` or directly to me at `errata@aceyourexams.net`.

An errata list will be posted at `http://errata.aceyourexams.net`. Check this errata list frequently.

Lesson 2

Parametric Distributions

Reading: Tse 2.2, Hogg, McKean, Craig 2.2,2.7

A *parametric distribution* is one that is defined by a fixed number of parameters. Examples of parametric distributions are the exponential distribution (parameter θ) and the Pareto distribution (parameters α, θ). Any distribution listed in the tables you receive at the exam is parametric.

It is traditional to use parametric distributions for claim counts (frequency) and loss size (severity). Parametric distributions have many advantages. One of the advantages of parametric distributions which makes them so useful for severity is that they handle inflation easily.

2.1 Transformations

You learn how to transform a distribution when you study probability. We will discuss the easiest case here.

If $Y = g(X)$, with $g(x)$ a one-to-one monotonically increasing function, then

$$F_Y(y) = \Pr(Y \le y) = \Pr\big(X \le g^{-1}(y)\big) = F_X\big(g^{-1}(y)\big) \tag{2.1}$$

and differentiating,

$$f_Y(y) = f_X\big(g^{-1}(y)\big)\frac{\mathrm{d}g^{-1}(y)}{\mathrm{d}y}$$

If $g(x)$ is one-to-one monotonically decreasing, then

$$F_Y(y) = \Pr(Y \le y) = \Pr\big(X \ge g^{-1}(y)\big) = S_X\big(g^{-1}(y)\big) \tag{2.2}$$

and differentiating,

$$f_Y(y) = -f_X\big(g^{-1}(y)\big)\frac{\mathrm{d}g^{-1}(y)}{\mathrm{d}y}$$

Putting both cases (monotonically increasing and monotonically decreasing) together:

$$f_Y(y) = f_X\big(g^{-1}(y)\big)\left|\frac{\mathrm{d}g^{-1}(y)}{\mathrm{d}y}\right| \tag{2.3}$$

Example 2A X follows a two-parameter Pareto distribution with parameters α and θ. You are given

$$Y = \ln\left(\frac{X}{\theta} + 1\right)$$

Determine the distribution of Y.

Solution:

$$\begin{aligned}
y &= \ln\left(\frac{x}{\theta}+1\right)\\
e^y - 1 &= \frac{x}{\theta}\\
x &= \theta(e^y-1)\\
F_Y(y) &= F_X\bigl(\theta(e^y-1)\bigr)\\
&= 1-\left(\frac{\theta}{\theta+\theta(e^y-1)}\right)^\alpha\\
&= 1-\left(\frac{\theta}{\theta e^y}\right)^\alpha\\
&= 1-e^{-\alpha y}
\end{aligned}$$

So Y's distribution is exponential with parameter $\theta = 1/\alpha$. □

We see in this example that an exponential can be obtained by transforming a Pareto.

2.2 Common parametric distributions

The tables provide a lot of information about the distributions, but if you don't recognize the distribution, you won't know to use the table. Therefore, it is a good idea to be familiar with the common distributions.

You should familiarize yourself with the *form* of each distribution, but not necessarily the constants. The constant is forced so that the density function will integrate to 1. If you know which distribution you are dealing with, you can figure out the constant. To emphasize this point, in the following discussion, we will use the letter c for constants rather than spelling out what the constants are. You are not trying to recognize the constant; you are trying to recognize the form.

We will mention the means and variances or second moments of the distributions. You need not memorize any of these. The tables give you the raw moments. You can calculate the variance as $\mathbf{E}[X^2]-\mathbf{E}[X]^2$. However, for frequently used distributions, you may want to memorize the mean and variance to save yourself some time when working out questions.

We will graph the distributions. You are not responsible for graphs, but they may help you understand the distributions.

The tables occasionally use the gamma function $\Gamma(x)$ in the formulas for the moments. You should have a basic knowledge of the gamma function; if you are not familiar with this function, see the sidebar. The tables also use the incomplete gamma and beta functions, and define them, but you can get by without knowing them.

2.2.1 Uniform

A uniform distribution has a constant density on $[d,u]$:

$$\begin{aligned}
f(x;d,u) &= \frac{1}{u-d} \qquad d\le x\le u\\
F(x;d,u) &= \begin{cases} 0 & x\le d\\ \dfrac{x-d}{u-d} & d\le x\le u\\ 1 & x\ge u\end{cases}
\end{aligned}$$

You recognize a uniform distribution both by its finite support[1] and by the lack of an x in the density function.

[1]"Support" is the range of values for which the probability density function is nonzero.

The gamma function

The gamma function $\Gamma(x)$ is a generalization to real numbers of the factorial function, defined by

$$\Gamma(x) = \int_0^\infty u^{x-1}e^{-u}\,\mathrm{d}u$$

For positive integers n,

$$\Gamma(n) = (n-1)!$$

The most important relationship for $\Gamma(x)$ that you should know is

$$\Gamma(x+1) = x\Gamma(x)$$

for any real number x.

Example 2B Evaluate $\dfrac{\Gamma(8.5)}{\Gamma(6.5)}$.

Solution:

$$\frac{\Gamma(8.5)}{\Gamma(6.5)} = \left(\frac{\Gamma(8.5)}{\Gamma(7.5)}\right)\left(\frac{\Gamma(7.5)}{\Gamma(6.5)}\right) = (7.5)(6.5) = \boxed{\mathbf{48.75}}$$

□

Its moments are

$$\mathbf{E}[X] = \frac{d+u}{2}$$
$$\mathrm{Var}(X) = \frac{(u-d)^2}{12}$$

Its mean, median, and midrange are equal. The best way to calculate the second moment is to add up the variance and the square of the mean. However, some students prefer to use the following easy-to-derive formula:

$$\mathbf{E}[X^2] = \frac{1}{u-d}\int_d^u x^2\,\mathrm{d}x = \frac{u^3-d^3}{3(u-d)} = \frac{u^2+ud+d^2}{3} \tag{2.4}$$

If $d = 0$, then the formula reduces to $u^2/3$.

The uniform distribution is not directly in the tables, so I recommend you memorize the formulas for mean and variance. However, if $d = 0$, then the uniform distribution is a special case of a beta distribution with $\theta = u$, $a = 1$, $b = 1$.

2.2.2 Beta

The probability density function of a beta distribution with $\theta = 1$ has the form

$$f(x; a, b) = cx^{a-1}(1-x)^{b-1} \qquad 0 \le x \le 1$$

The parameters a and b must be positive. They may equal 1, in which case the corresponding factor is missing from the density function. Thus if $a = b = 1$, the beta distribution is a uniform distribution.

You recognize a beta distribution both by its finite support—it's the only common distribution for which the density is nonzero only on a finite range of values—and by factors with x and $1-x$ raised to powers and no other use of x in the density function.

If θ is arbitrary, then the form of the probability density function is

$$f(x; a, b, \theta) = cx^{a-1}(\theta-x)^{b-1} \qquad 0 \le x \le \theta$$

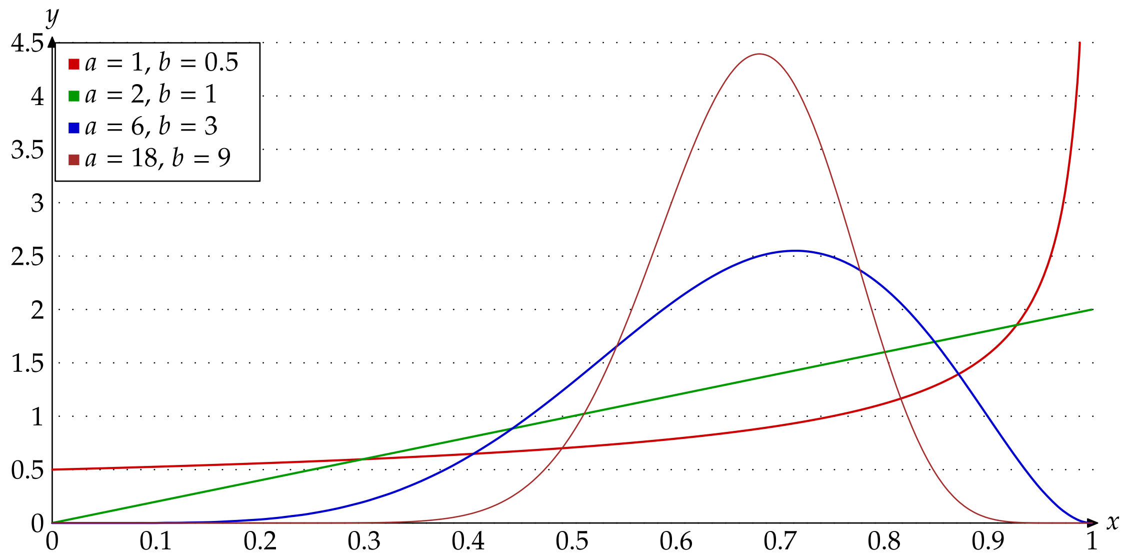

Figure 2.1: Probability density function of four beta distributions with $\theta = 1$ and mean 2/3

The distribution function can be evaluated if a or b is an integer. The moments are

$$\mathbf{E}[X] = \frac{\theta a}{a+b}$$
$$\operatorname{Var}(X) = \frac{\theta^2 ab}{(a+b)^2(a+b+1)}$$

The mode is $\theta(a-1)/(a+b-2)$ when a and b are both greater than 1, but you are not responsible for this fact.

Figure 2.1 graphs four beta distributions with $\theta = 1$ all having mean $2/3$. You can see how the distribution becomes more peaked and normal looking as a and b increase.

2.2.3 Exponential

The probability density function of an exponential distribution has the form

$$f(x;\theta) = ce^{-x/\theta} \qquad x \geq 0$$

θ must be positive.

You recognize an exponential distribution when the density function has e raised to a multiple of x, and no other use of x.

The distribution function is easily evaluated. The moments are:

$$\mathbf{E}[X] = \theta$$
$$\operatorname{Var}(X) = \theta^2$$

Figure 2.2 graphs three exponential distributions. The higher the parameter, the more weight placed on higher numbers.

The amount of time between events in a Poisson process is exponentially distributed. We'll discuss Poisson processes later in the course.

2.2.4 Weibull

A Weibull distribution is a transformed exponential distribution. If Y is exponential with mean μ, then $X = Y^{1/\tau}$ is Weibull with parameters $\theta = \mu^{1/\tau}$ and τ. An exponential is a special case of a Weibull with $\tau = 1$.

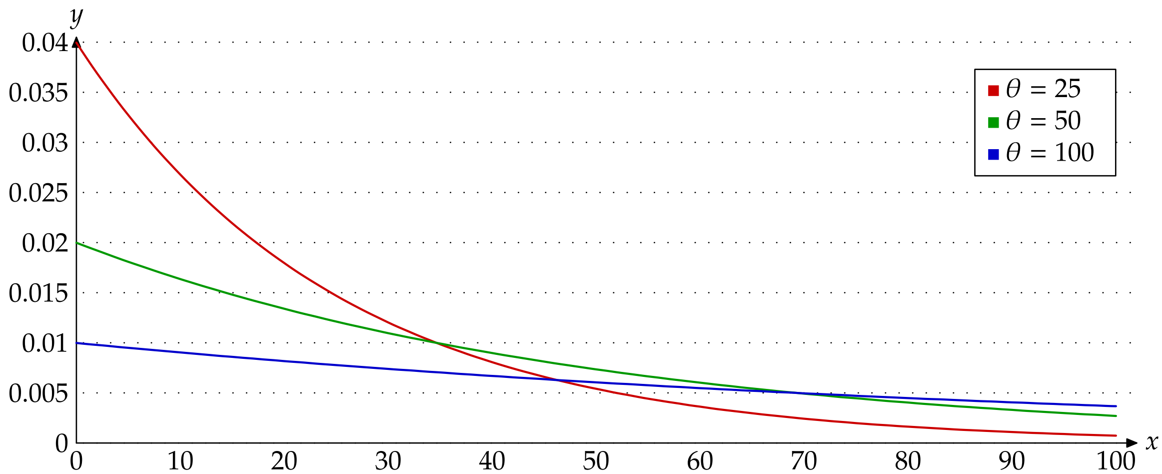

Figure 2.2: Probability density function of three exponential distributions

The form of the density function is

$$f(x;\tau,\theta) = cx^{\tau-1}e^{-(x/\theta)^\tau} \qquad x \ge 0$$

Both parameters must be positive. The shape parameter is τ and the scale parameter is θ.

You recognize a Weibull distribution when the density function has e raised to a multiple of a power of x, and in addition has a corresponding power of x, one lower than the power in the exponential, as a factor.

When $\theta = 1$, we say that the distribution is a *standard* Weibull.

The distribution function is easily evaluated, but the moments require evaluating the gamma function, which usually requires numerical techniques. The moments are

$$\mathbf{E}[X] = \theta\Gamma(1 + 1/\tau)$$
$$\mathbf{E}[X^2] = \theta^2\Gamma(1 + 2/\tau)$$

Figure 2.3 graphs three Weibull distributions with mean 50. The distribution has a non-zero mode when $\tau > 1$. Notice that the distribution with $\tau = 0.5$ puts a lot of weight on small numbers. To make up for this, it will also have to put higher weight than the other two distributions on very large numbers, so although it's not shown, its graph will cross the other two graphs for high x

2.2.5 Gamma

The form of the density function of a gamma distribution is

$$f(x;\alpha,\theta) = cx^{\alpha-1}e^{-x/\theta} \qquad x \ge 0$$

Both parameters must be positive. The shape parameter is α and the scale parameter is θ.

When α is an integer, a gamma random variable with parameters α and θ is the sum of α independent exponential random variables with parameter θ. In particular, when $\alpha = 1$, the gamma random variable is exponential. The gamma distribution is called an Erlang distribution when α is an integer.

You recognize a gamma distribution when the density function has e raised to a multiple of x, and in addition has x raised to a power. Contrast this with a Weibull, where e is raised to a multiple of a *power* of x.

The distribution function may be evaluated if α is an integer; otherwise numerical techniques are needed. However, the moments are easily evaluated:

$$\mathbf{E}[X] = \alpha\theta$$
$$\mathrm{Var}(X) = \alpha\theta^2$$

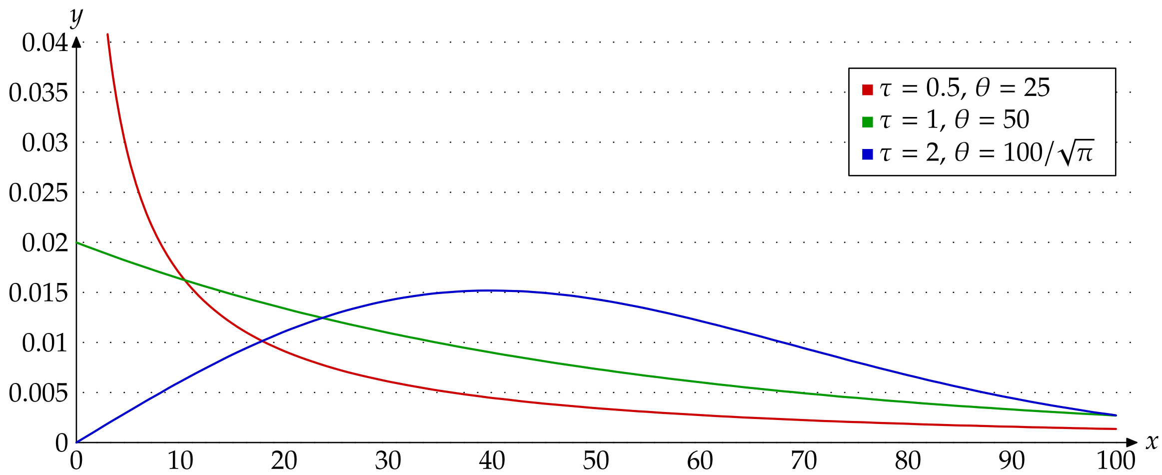

Figure 2.3: Probability density function of three Weibull distributions with mean 50

Figure 2.4 graphs three gamma distributions with mean 50. As α goes to infinity, the graph's peak narrows and the distribution converges to a normal distribution.

The gamma distribution is one of the few for which the moment generating function has a closed form. In particular, the moment generating function of an exponential has a closed form. The only other distributions in the tables with closed form moment generating functions are the normal distribution (not actually in the tables, but the formula for the lognormal moments is the MGF of a normal) and the inverse Gaussian.

2.2.6 Pareto

When we say "Pareto", we mean a *two-parameter* Pareto. On recent exams, they write out "two-parameter" to make it clear, but on older exams, you will often find the word "Pareto" with no qualifier. It always refers to a two-parameter Pareto, not a single-parameter Pareto.

The form of the density function of a two-parameter Pareto is

$$f(x) = \frac{c}{(\theta + x)^{\alpha+1}} \qquad x \ge 0$$

Both parameters must be positive.

You recognize a Pareto when the density function has a denominator with x plus a constant raised to a power.

The distribution function is easily evaluated. The moments are

$$\begin{aligned} \mathbf{E}[X] &= \frac{\theta}{\alpha - 1} & \alpha > 1 \\ \mathbf{E}[X^2] &= \frac{2\theta^2}{(\alpha-1)(\alpha-2)} & \alpha > 2 \end{aligned}$$

When α does not satisfy these conditions, the corresponding moments don't exist.

A shortcut formula for the variance of a Pareto is

$$\mathrm{Var}(X) = \mathbf{E}[X]^2 \left(\frac{\alpha}{\alpha - 2}\right)$$

Figure 2.5 graphs three Pareto distributions, one with $\alpha < 1$ and the other two with mean 50. Although the one with $\alpha = 0.5$ puts higher weight on small numbers than the other two, its mean is infinite; it puts higher weight on large numbers than the other two, and its graph eventually crosses the other two as $x \to \infty$.

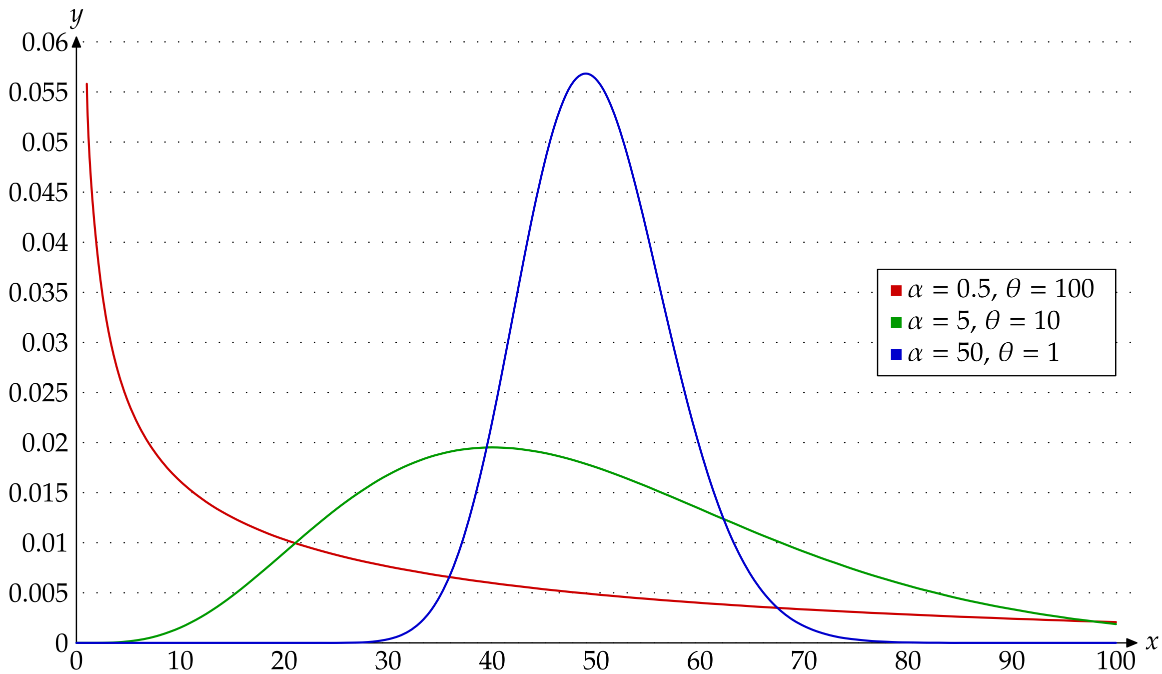

Figure 2.4: Probability density function of three gamma distributions with mean 50

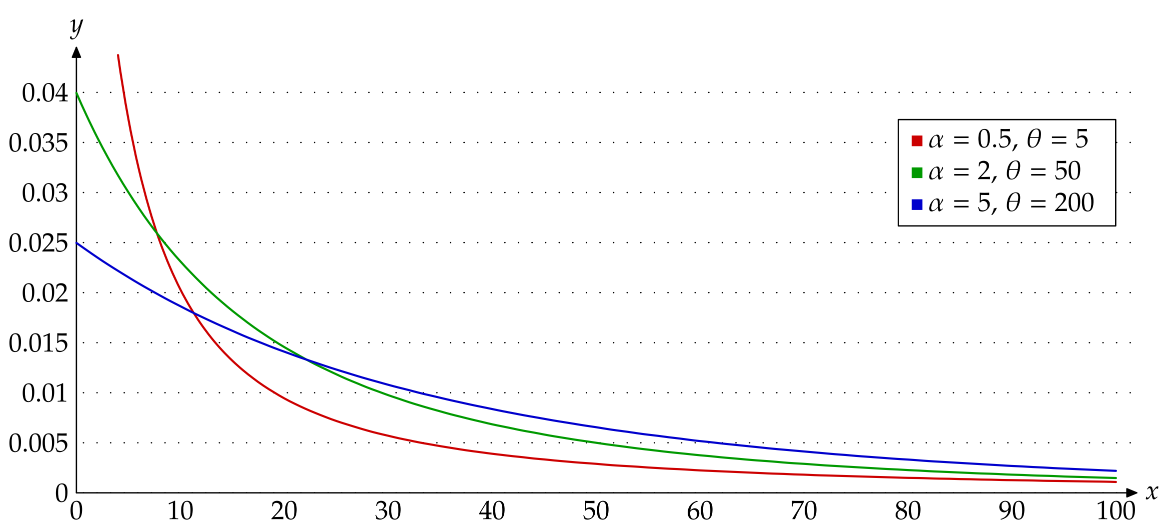

Figure 2.5: Probability density function of three Pareto distributions

2.2.7 Single-parameter Pareto

The form of the density function of a single-parameter Pareto is

$$f(x) = \frac{c}{x^{\alpha+1}} \qquad x \geq \theta$$

α must be positive. θ is not considered a parameter since it must be selected in advance, based on what you want the range to be.

You recognize a single-parameter Pareto by the range of nonzero values for its density function—unlike most other distributions, this range does not start at 0—and by the form of the density function, which has a denominator with x raised to a power. A beta distribution may also have x raised to a negative power, but its density function is 0 above a finite number.

A single-parameter Pareto X is a two-parameter Pareto Y shifted by θ: $X = Y + \theta$. Thus it has the same variance, and the mean is θ greater than the mean of a two-parameter Pareto with the same parameters.

$$\mathbf{E}[X] = \frac{\alpha\theta}{\alpha - 1} \qquad \alpha > 1$$
$$\mathbf{E}[X^2] = \frac{\alpha\theta^2}{\alpha - 2} \qquad \alpha > 2$$

2.2.8 Lognormal

The form of the density function of a lognormal distribution is

$$f(x) = \frac{ce^{-(\ln x-\mu)^2/2\sigma^2}}{x} \qquad x > 0$$

σ must be nonnegative.

You recognize a lognormal by the $\ln x$ in the exponent.

If Y is normal, then $X = e^Y$ is lognormal with the same parameters μ and σ. Thus, to calculate the distribution function, use

$$F_X(x) = F_Y(\ln x) = \Phi\left(\frac{\ln x - \mu}{\sigma}\right)$$

where $\Phi(x)$ is the standard normal distribution function, for which you are given tables. The moments of a lognormal are

$$\mathbf{E}[X] = e^{\mu+0.5\sigma^2}$$
$$\mathbf{E}[X^2] = e^{2\mu+2\sigma^2}$$

More generally, $\mathbf{E}[X^k] = \mathbf{E}[e^{kY}] = M_Y(k)$, where $M_Y(k)$ is the moment generating function of the corresponding normal distribution.

Figure 2.6 graphs three lognormals with mean 50. The mode is $\exp(\mu - \sigma^2)$, as stated in the tables. For $\mu = 2$, the mode is off the graph. As σ gets lower, the distribution flattens out.

Table 2.1 is a summary of the forms of probability density functions for common distributions.

Exercises

2.1. X follows an exponential distribution with mean 10.

Determine the mean of X^4.

Exercises continue on the next page . . .

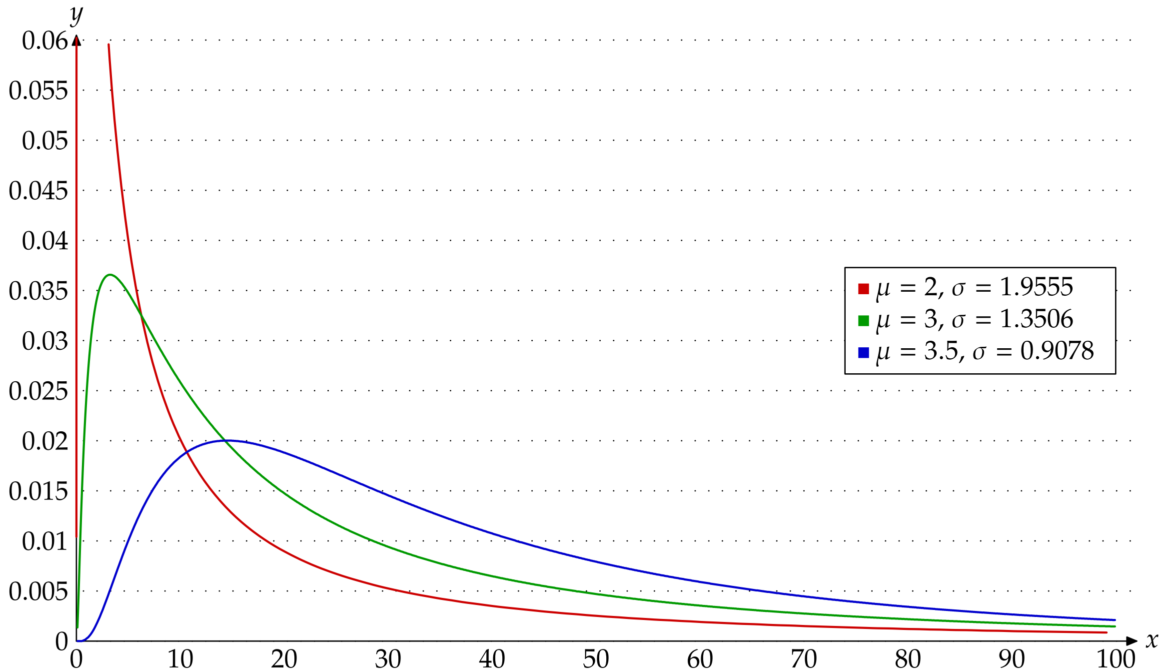

Figure 2.6: Probability density function of three lognormal distributions with mean 50

Table 2.1: Forms of probability density functions for common distributions

Distribution	Probability density function	
Uniform	c	$d \leq x \leq u$
Beta	$cx^{a-1}(\theta - x)^{b-1}$	$0 \leq x \leq \theta$
Exponential	$ce^{-x/\theta}$	$x \geq 0$
Weibull	$cx^{\tau-1}e^{-x^\tau/\theta^\tau}$	$x \geq 0$
Gamma	$cx^{\alpha-1}e^{-x/\theta}$	$x \geq 0$
Pareto	$\dfrac{c}{(x+\theta)^{\alpha+1}}$	$x \geq 0$
Single-parameter Pareto	$\dfrac{c}{x^{\alpha+1}}$	$x \geq \theta$
Lognormal	$\dfrac{ce^{-(\ln x-\mu)^2/2\sigma^2}}{x}$	$x > 0$

Exercises continue on the next page . . .

2.2. You are given

- X is exponential with mean 2.
- $Y = X^{1.5}$.

Calculate $\mathbf{E}[Y^2]$.

2.3. X follows a gamma distribution with parameters $\alpha = 2.5$ and $\theta = 10$.

$Y = 1/X$.

Calculate $\mathrm{Var}(Y)$.

2.4. **[CAS3-F05:19]** Claim size, X, follows a Pareto distribution with parameters α and θ. A transformed distribution, Y, is created such tht $Y = X^{1/\tau}$.

Determine the probability density function of Y.

A. $\dfrac{\tau\theta y^{\tau-1}}{(y+\theta)^{\tau+1}}$ B. $\dfrac{\alpha\theta^{\alpha}\tau y^{\tau-1}}{(y^{\tau}+\theta)^{\alpha+1}}$ C. $\dfrac{\theta\alpha^{\theta}}{(y+\alpha)^{\theta+1}}$ D. $\dfrac{\alpha\tau(y/\theta)^{\tau}}{y\bigl(1+(y/\theta)^{\tau}\bigr)^{\alpha+1}}$ E. $\dfrac{\alpha\theta^{\alpha}}{(y^{\tau}+\theta)^{\alpha+1}}$

2.5. **[CAS3-S06:27]** The following information is available regarding the random variables X and Y:

- X follows a Pareto distribution with $\alpha = 2$ and $\theta = 100$.
- $Y = \ln\bigl(1 + (X/\theta)\bigr)$

Calculate the variance of Y.

A. Less than 0.10
B. At least 0.10, but less than 0.20
C. At least 0.20, but less than 0.30
D. At least 0.30, but less than 0.40
E. At least 0.40

Additional released exam questions: MAS-I F18:24

Solutions

2.1. The k^{th} moment for an exponential is given in the tables:

$$\mathbf{E}[X^k] = k!\theta^k$$

for $k = 4$ and the mean $\theta = 10$, this is $4!(10^4) = \boxed{\mathbf{240{,}000}}$.

2.2. While Y is Weibull, you don't need to know that. It's easier to use $Y^2 = X^3$ and look up the third moment of an exponential.

$$\mathbf{E}[X^3] = 3!\theta^3 = 6(2^3) = \boxed{\mathbf{48}}$$

2.3. We calculate $\mathbf{E}[Y]$ and $\mathbf{E}[Y^2]$, or $\mathbf{E}[X^{-1}]$ and $\mathbf{E}[X^{-2}]$. Note that the special formula in the tables for integral moments of a gamma, $\mathbf{E}[X^k] = \theta^k(\alpha+k-1)\cdots\alpha$ only applies when k is a *positive* integer, so it cannot be used for the -1 and -2 moments. Instead, we must use the general formula for moments given in the tables,

$$\mathbf{E}[X^k] = \frac{\theta^k\Gamma(\alpha+k)}{\Gamma(\alpha)}$$

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